Fundamenta Informaticae xxx (2013) 1–16
DOI 10.3233/FI-2013-xx
IOS Press

Proving Propositional Tautologies in a Natural Dialogue

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Abstract. The paper proposes a dialogue system LND which brings together and unifies two traditions in studying dialogue as a game: the dialogical logic introduced by Lorenzen; and persuasion dialogue games as specified by Prakken. The first approach allows the representation of formal dialogues in which the validity of argument is the topic discussed. The second tradition has focused on natural dialogues examining, e.g., informal fallacies typical in real-life communication. Our goal is to unite these two approaches in order to allow communicating agents to benefit from the advantages of both, i.e., to equip them with the ability not only to persuade each other about facts, but also to prove that a formula used in an argument is a classical propositional tautology. To this end, we propose a new description of the dialogical logic which meets the requirements of Prakken's generic specification for natural dialogues, and we introduce rules allowing to embed a formal dialogue in a natural one. We also show the correspondence result between the original and the new version of the dialogical logic, i.e., we show that a winning strategy for a proponent in the original version of the dialogical logic means a winning strategy for a proponent in the new version, and conversely.

1. Introduction

The paper proposes a dialogue system LND which brings together and unifies two traditions in studying dialogue as a game: the dialogical logic introduced by Lorenzen [10, 11, 16]; and persuasion dialogue games as specified by Prakken [15]. The first approach allows the representation of *formal dialogues*, i.e., dialogues in which the validity of argument is the topic discussed. The second tradition has focused on *natural dialogues*, i.e., dialogues typical for the real-life practice in which agents communicate in a natural language and talk about the facts such as e.g. the safety of someone's car.

There are at least two important features of the second tradition. In systems for natural dialogues, communicating agents are assumed to exchange not propositions, but rather speech acts (see e.g. [1, 17]). This assumption is common for modelling dialogue protocols in multi-agent systems, mainly due to the influential work by Walton and Krabbe [20]. In communication languages such as KQML and FIPA speech acts are used to express intentions of their performers and are specified by means of agent's mental attitudes. The most important issue of logical modelling of communication in teamwork, especially during planning is studied in [5]. This work provides a schema of deliberation dialogue along with semantics of adequate speech acts. An implementation of speech acts in a paraconsistent framework is shown in [4]. In this approach, a natural four-valued model of interaction is based on 4QL formalism [13]. The second important feature of systems for natural dialogues is that, following Hamblin's program [7], their primary aim is to design a game which does not allow agents to commit fallacies such as circularity in reasoning. This approach has resulted in many formal systems exploring different informal fallacies (see e.g. [12, 22]).

In real-life communication, however, the speakers commit both informal and formal fallacies [3, 21]. A formal fallacy is understood as an argument which is invalid according to some logical system. Amongst fallacies which do not follow the rules of classical propositional logic and are claimed to be common in natural dialogues are, e.g., fallacies of incorrect operations on implication, i.e. *denying the antecedent* ($\varphi \rightarrow \psi$, $\neg\varphi$, therefore $\neg\psi$) and *affirming the consequent* ($\varphi \rightarrow \psi$, ψ, therefore φ). A system aiming to disallow the execution of both informal and formal fallacies is proposed by pragma-dialectics [6]. According to *rule 6* of the critical discussion system [6, p.144], the antagonist may not only challenge the propositional content of premises used by the protagonist, but also the justificatory force (i.e., validity) of his reasoning. The pragma-dialectical system requires that the protagonist has to use rules of some logic to defend his standpoint, still it does not provide a formal account of dialogues.

The attempt to formally describe the system of critical discussion was made in [19], however, since a protocol is not fully specified there, it is not possible to evaluate whether and how the proposed dialogue system actually allows formal fallacies to be dealt with. On the other hand, Walton and Krabbe [20] propose two dialogical systems: PPD, which describes natural persuasion dialogues, and RPD, which describes formal dialogues in the style of the dialogical logic. They show how to embed RPD-game in PPD-game, however, the aim of embedding is not to prove whether a formula is a tautology or to identify formal fallacies committed during a natural dialogue. The aim is to allow an agent to help his opponent to infer the proper conclusions, i.e. conclusions logically following from the premises. Say that an agent during a dialogue commits himself to both $\varphi \rightarrow \psi$, and $\neg\psi$. The rules of PPD system does not require that the agent should, in consequence, commit himself to $\neg\varphi$. Yet, his opponent can start the RPD-game in which he will be able to convince the agent that he should accept $\neg\varphi$ on the basis of his previous commitments and the rules of propositional logic. As a result, an agent does not prove which arguments are valid or does not learn what formulas are propositional tautologies, but he learns that the fact $\neg\varphi$ holds. In many other contemporary dialogue games, it is impossible to perform an invalid argument at all, since they have a logic as a part of the system itself.

The aim of this paper is to make a first key step in including formal fallacies in the formal study of natural dialogues. We propose a dialogue system, LND, that allows communicating agents to prove that a formula used in an argument is a *classical propositional tautology*, and, as a result, to identify and eliminate *classical propositional formal fallacies* committed during a natural dialogue. To this end, we need to combine a system for representing natural dialogues with a system for representing formal dialogues. In the first case, we use the framework proposed by Prakken [15], since it provides a generic and formal specification of the main elements of dialogue systems for persuasion. For handling formal fallacies in a dialogue, we use the dialogical logic introduced by Lorenzen [10, 11, 16]. His dialogue games allow the players to prove that a formula is a tautology of classical propositional logic, if the proponent has a winning strategy in a given game.[1] Note that the aim of this system is not to jointly build an argument: φ, therefore ψ, as in inquiry dialogues (see e.g. [2]), but to allow the participants to play against each other starting with opposing viewpoints on an argument validity and determining which player wins.

The key challenge here is that the dialogical logic communication language and structure are different from systems for natural dialogues. For example, in Lorenzen's system the only moves available to speakers are: X *attacks* φ and X *defends* φ, while, according to Prakken's specification, in systems for natural dialogues the legal locutions are: *claim* φ, *why* φ, *concede* φ, *retract* φ, φ *since* S, *question* φ. The main contribution of this paper is to introduce a new description of the dialogical logic which meets the requirements of Prakken's generic specification, and to propose rules allowing for embedding a formal dialogue in a natural one. We also show the correspondence result between the original and the new version of the dialogical logic, i.e., we show that a winning strategy for a proponent in the original version of the dialogical logic means a winning strategy for a proponent in the new one, and conversely.

The paper is organized as follows. Section 2 introduces an example which illustrates the motivation for the dialogue system LND proposed in the paper. In Sections 3 and 4, we give a brief overview of Prakken's generic specification for systems of natural dialogues, and Lorenzen's dialogical logic, respectively. In Section 5, we propose a new description of the dialogical logic in accordance with the generic specification for natural dialogues. In Section 6, the rules for embedding formal dialogues in natural dialogues are introduced. Finally, Section 7 presents the correspondence result between the original and the new version of the dialogical logic.

2. Motivation example

To show the motivation behind our research, let us consider the persuasion dialogue given in [14] in which Paul and Olga discuss whether or not a car which has an airbag is safe. We modify the dialogue to illustrate the main idea of our approach.

(1) Paul: If a car doesn't have an airbag then it isn't safe. (stating a claim)
(2) Olga: That is true. (conceding a claim)
(3) Paul: My car is safe. (making a claim)
(4) Olga: Why is your car safe? (asking grounds for a claim)
(5) Paul: My car has an airbag and if a car doesn't have an airbag then it isn't safe, thus my car is safe. (making an argument, committing a formal fallacy)
(6) Olga: Why do you think that your reasoning is correct? (asking grounds for the argument)
(7) Paul: It is correct because the scheme $p \wedge (\neg p \to \neg q) \to q$ is a tautology. (offering grounds for the argument, expressing the commitment to a formal fallacy)
(8) Olga: No it isn't. (stating a counterclaim)

⋮

[1]There are many versions of the dialogical logic which show the analogical results for other logical accounts.

(18) Paul: OK, you are right. I was wrong that the scheme $p \wedge (\neg p \rightarrow \neg q) \rightarrow q$ is a tautology. (retracting from a formal fallacy)
In the dialogue, Paul supported his reasoning by a formula which in his opinion was a tautology of classical propositional logic. Olga questioned this and after continuing a discussion for some more time Paul changed his mind. Why did he retract? To answer this question, assume that Paul and Olga attend a course of logic and try to examine the validity of the formula. Their dialogue could be as follows:
(9) Paul: Let's start a Lorenzen game. I'll show you that the implication $p \wedge (\neg p \rightarrow \neg q) \rightarrow q$ is valid. (initializing the game by claiming a formula)
(10) Olga: I'll help you and assume that $p \wedge (\neg p \rightarrow \neg q)$ is true. (attacking implication by stating its antecedent)
(11) Paul: Is p true? (attacking conjunction)
(12) Olga: Yes, it is. (defending conjunction)
(13) Paul: Is $\neg p \rightarrow \neg q$ true? (attacking conjunction again)
(14) Olga: Yes, it is. (defending conjunction by stating implication)
(15) Paul: Why is $\neg p \rightarrow \neg q$ true? Can you show this? I'll help you and assume $\neg p$. (attacking implication by stating its antecedent)
(16) Olga: Thus, I state that $\neg q$ is true. (defending implication by stating its consequent)
(17) Paul: Well, I have no more available moves. Game is over. (ending the game)

In the first part of the dialogue (the moves 1–8) played according to dialogue systems such as [14], Paul's reasoning performed in the fifth move was based on a fallacy of denying the antecedent. This mistake was then identified during the discussion. In the move 18, Paul retracts from the commitment to the formal fallacy. Thus, this dialogue is an example of natural communication in which a formal fallacy was recognized and eliminated. A dialogue aiming to eliminate this mistake is presented in the second part of the example (the moves 9–17) which implements the idea of the dialogical logic of Lorenzen [11].

3. A specification of systems for natural dialogues

Formal systems for a natural dialogue aim to formally model different dialogical phenomena, such as, e.g., informal fallacies which are typical in real-life communication (see e.g. [7, 12, 22]). In [15], Prakken presents a general specification of common elements of such systems. In this section, we summarize the most important components.

3.1. Key elements of dialogue systems

Every dialogue system has a *dialogue purpose*, a set A of *participants* and a set R of *roles* which participants can adopt during a game. Contents of utterances used by players in dialogues are expressed in a *topic language* L_t. At the beginning of a dialogue every player s has assigned a (possible empty) set of *commitments* $C_s \subseteq L_t$ which usually changes during a dialogue. Every dialogue system includes a logic L consisting of a topic language L_t and a set R of inference rules over L_t.

The dialogue system consists of several sets of rules. First, the communication language L_c defines **locution rules** which describe which type of speech acts players can execute during a dialogue. For example, in a dialogue which is played according to some dialogue system, agent *i* may be allowed to use only the following speech acts: *claim* φ for asserting proposition φ; *concede* φ for agreeing with the opponent about φ; *why* φ for challenging φ; φ *since* ψ for supporting the conclusion φ with the premise ψ; and *question* φ for asking whether the opponent accepts that φ holds. The central element of a dialogue system is its **protocol** which determines the interaction between locutions. In other words, it specifies which locution can be performed as a reply to another locution. Let M be a set of *moves*. The set of *finite dialogues* $M^{<\infty}$ is the set of all finite sequences $m_1, \ldots, m_i$ from M. A *protocol*, specifying the legal moves at each stage of a dialogue, is a function $P : Pow(L_t) \times D \to Pow(L_c)$ where $D \subseteq M^{<\infty}$. The elements of D are called the *legal finite dialogues*. For example, after *question* φ the opponent can either perform *claim* φ or *claim* $\neg\varphi$. A set of **effect rules** for L_c (formally, $C_i : M^{<\infty} \to Pow(L_t)$) specifies for each utterance $\varphi \in L_c$ the effects which this locution makes on a set of commitments of the participant i (a commitment of i is a sentence that i publicly declared as his belief). The function C_i for a sequence of moves assigns a set of commitments. For example, the sequence of moves ending with the performance of *claim* φ by agent *i* results in adding the proposition φ into i's commitment base.

In some of the dialogue systems, the protocol is enriched with rules regulating turntaking, termination and the outcome of a dialogue. **Turntaking rules** determine a turn of a dialogue i.e., a maximal sequence of moves in which the same player is to move. A *turntaking function* T is a function $T : M^{<\infty} \to Pow(A)$. **Termination rules** determine the cases where no move is legal. They should specify the conditions under which the protocol returns the empty set. **Outcome rules** define the outcome of a dialogue, i.e. which player wins and which player lose the dialogue (for more details about the specification of natural dialogues see [15]).

3.2. Example

According to the specification for natural dialogues, the beginning of the dialogue between Paul and Olga can be formally represented as follows. Paul starts the dialogue making a claim to which Olga agrees:

P$_1$**:** *claim* (¬ airbag → ¬ safe)
O$_2$**:** *concede* (¬ airbag → ¬ safe)
P$_3$**:** *claim* (safe)
O$_4$**:** *why* (safe)
P$_5$**:** safe *since* {airbag; ¬ airbag → ¬ safe}
O$_6$**:** *why* (airbag ∧ (¬ airbag → ¬ safe) → safe)
P$_7$**:** airbag ∧ (¬ airbag → ¬ safe) → safe *since* {the formula $p \wedge (\neg p \to \neg q) \to q$ is a tautology}
O$_8$**:** *claim* (¬ the formula $p \wedge (\neg p \to \neg q) \to q$ is a tautology)

As a response to Olga's challenge O_4, Paul shows in the move P_5 how he obtained this claim, (safe), i.e. he concluded it on the basis of two assumptions: his car has an airbag; and if a car does not have an airbag, then it is not safe. The formal reconstruction of this move is designed in such a way that allows the embedding of the dialogue about argument validity in Prakken's protocol. Specifically, the implication with the antecedent being the conjunction of premises and the consequent being a conclusion can be explicitly challenged by the opponent (O_6). As a response, the proponent may defend the implication by stating that its formalization is the propositional tautology (P_7). Then, Olga disagrees with Paul about the validity of the implication (O_8).

Now, Paul and Olga will initiate a new (embedded) dialogue whose goal will be to decide whether the formula under discussion is a tautology of classical propositional calculus and then they will resolve the conflict of opinion. This dialogue cannot be expressed in the formalism of natural dialogue systems directly. To describe it, we use a system for formal dialogues which was introduced by Lorenzen.

4. A system for formal dialogues

Now, we briefly describe a system for formal dialogues called dialogical logic (DL) introduced by Lorenzen [11] and further extended and improved (see e.g. [9, 10, 16]). In this approach, dialogues are treated as games in which two parties, opponent and proponent, examine a formula. Their goal is to verify whether this formula is valid (we focus only on the validity of classical propositional logic). A game proceeds according to the set of rules which ensure that the formula is valid iff the proponent has a winning strategy.

4.1. Structural and particle rules of dialogical logic

In this section, main elements of the dialogical logic (DL) for classical propositional logic are presented (see [10, 11, 16]). In each DL-game, there are two players involved: *proponent* of the main formula (**P**) and *opponent* of this formula (**O**). During the game they make use of two types of moves: they *attack* or *defend* a formula. The dialogical logic is specified by two kinds of rules: particle rules describing the way a formula can be attacked and defended depending on its main connective and structural rules determining the general organization of the game.

Particle rules for basic propositional language are presented in Table 1.[2] They are divided into rules for attacks: **PR-1a** - **PR-4a** and rules for defences **PR-1d** - **PR-4d**. According to the rule **PR-1a**, a negation $\neg A$ is attacked by asserting A. There is no defence of $\neg A$ available (rule **PR-1d**). If a player attacks $A \wedge B$, he attacks the first or the second conjunct (rule **P2a**). The defence of $A \wedge B$ is to state the attacked conjunct (rule **P2d**). If a player attacks $A \vee B$, he performs the move "?" which questions the whole disjunction (rule **PR-3a**). If he defends $A \vee B$, he asserts any element of the attacked disjunction (rule **PR-3d**). Finally, if a player attacks $A \rightarrow B$, he asserts the antecedent (**PR-4a**) and if he defends the implication, he asserts the consequent (rule **PR-4d**).

			a (attack)	**d** (defence)
PR-1	negation	$\neg A$	A	—-
PR-2	conjunction	$A \wedge B$	1?	A
			2?	B
PR-3	disjunction	$A \vee B$	?	A
				B
PR-4	implication	$A \rightarrow B$	A	B

Table 1. Particle rules for the basic propositional language

Structural rules define what sequences of attacks and defences count as dialogues. A *dialogue* for a formula (the thesis) A, **D**(A), is a set of dialogue games consisting of sequences of moves. Depending on which player makes the move, we talk about **P**-*statement* and **O**-*statement*.

[2]Original version of the dialogue logic was meant for intuitionistic propositional logic and then it has been extended to the classical case.

Structural rules for classical propositional logic are specified as follows:
SR-0: Starting Rule. For any $\triangle$ $\in$**D**(A), the thesis has position 0. At even positions **P** makes a move, and at odd positions it is **O** who moves.
SR-1$_c$: Classical Round Closure Rule. Whenever player **X** is to play, he can attack any move of **Y** in so far as the other rules let him do so, or he can defend against any attack of **Y**.
SR-2: Branching Rule For Dialogical Games. Any game situation were **O** is to play and has to choose between several moves will generate a distinct game for every *propositional choice* available to **O** (for more details see [10, 16]).[3]
SR-3: Winning Rule For Dialogical Games. A dialogical game $\triangle$ $\in$**D**(A) is said to be closed iff there is some atomic formula which has been played by both players. A dialogue game is finished iff it is closed or the rules do not allow any further move by the player who has to move. Let $\triangle$ be a finished game. If $\triangle$ is closed, **P** wins it, otherwise he loses it.
SR-4: Shifting Rule. **O** cannot switch to another game before the game he is playing is closed.
SR-5 Formal Use of Atomic Formulas. An atomic formula is introduced by a move if it has not been played in a previous move of the game. **P** cannot introduce atomic formulas (i.e. he can use an atomic formula iff **O** introduced it in a previous move). Atomic formulas cannot be attacked.
SR-6$_c$: Classical No-Delaying-Tactics Rule. No strict repetition is allowed (for more details see [10, 16]).

All dialogue games create a *tree* of which the root is constituted by the initial thesis of the dialogue. **P** cannot win **D**(A) unless he is able to win all the games belonging to **D**(A).

Definition 4.1. A propositional formula A is said to be *dialogically valid* if all games belonging to the dialogue **D**(A) are closed.

In [18] Stegmüller proved that the dialogical definition of validity coincides with the standard definition of validity. Therefore, a propositional formula A is a propositional tautology if **P** can win **D**(A).

4.2. Example

The continuation of the dialogue game between Paul and Olga can be described using the DL-rules. At the beginning Paul, taking the role of the proponent, proposes a formula and states that it is true:
$\mathbf{P}_9$: $p \wedge (\neg p \rightarrow \neg q) \rightarrow q$ (Paul as a proponent makes initial statement of the formula of which validity will be checked during the dialogue; see **SR-0**)
$\mathbf{O}_{10}$: $p \wedge (\neg p \rightarrow \neg q)$ (Olga as an opponent attacks the implication in $\mathbf{P}_9$ by stating that its antecedent is true; see **PR-4a**)
$\mathbf{P}_{11}$: p? (Paul attacks the conjunction from $\mathbf{O}_{10}$ by asking whether its first element is true; see **PR-2a**)
$\mathbf{O}_{12}$: p (Olga defends the conjunction attacked in $\mathbf{P}_{11}$ by stating that the element questioned by Paul is true; see **PR-2d**)
$\mathbf{P}_{13}$: $(\neg p \rightarrow \neg q)$? (Paul again attacks the conjunction in $\mathbf{O}_{10}$, this time by asking whether its second element is true; see **PR-2a**)
$\mathbf{O}_{14}$: $\neg p \rightarrow \neg q$ (Olga defends the conjunction by stating that the questioned element is true; see **PR-2d**)

[3] A propositional choice for **O** is when he creates distinct games in order to: (i) defend a disjunction, (ii) attack a conjunction, or (iii) react to an attack against an implication.

$\mathbf{P}_{15}$: $\neg p$ (Paul attacks the implication from $\mathbf{O}_{14}$ by stating that its antecedent is true; see **PR-4a**)
$\mathbf{O}_{16}$: $\neg q$ (Olga defends the implication from $\mathbf{O}_{14}$ by stating that its consequent is true; see **PR-4d**)

The only way to attack the negation $\neg q$ is to say q. Paul cannot perform such a move since as a proponent he cannot state an atomic formula (see **SR-5**). Therefore, there is no legal move for Paul and the dialogue finishes. The last legal move was performed by Olga and she is the winner of the game (see **SR-3**), which means that the initial formula is not a tautology of classical propositional logic. Paul must retract from the commitment that the formula is a tautology. As a result, the fallacy of denying the antecedent committed by Paul in P_5 is identified and his commitment to this formal fallacy from P_7 is eliminated. Yet, the DL-communication language differs from the language specified by Prakken. In order to embed this fragment of Paul and Olga's dialogue into a natural dialogue without the necessity of changing the communication language, we need to describe the DL system in terms of Prakken's specification for dialogue systems.

5. A new specification for dialogical logic

The paper proposes a new dialogue system LND which allows for the elimination of formal fallacies committed during a natural dialogue. This means that the system has to describe how to embed a formal dialogue (Lorenzen-like) game in a natural dialogue (Prakken-like) game. Such an aim requires to first unite and unify the representation of natural and formal dialogues. This section introduces the part of LND regulating formal dialogue games and shows how the dialogical logic DL could be translated to the specification of natural dialogue systems described in Section 3.

5.1. Locution rules

In this paper, a *dialogue goal* of Lorenzen-like games is limited to the verification of validity in classical propositional logic. The *set of players* consists of two elements $\{\mathbf{O}, \mathbf{P}\}$. *Topic language* L_t is assumed here to be that of classical propositional logic. The *communication language* in dialogical logic may seem to be, at the first glance, very limited, since it consists only of two types of actions: *attack* and *defence*. The careful reconstruction of these actions reveals, however, that depending on the structure of the attacked or defended formula, those actions can be mapped into various locutions considered by Prakken. The locution rules for formal dialogue games in LND are as follows:

L1 Claim *claim* φ is performed when a player: (1) attacks $\neg A$, then φ is a formula A, (2) defends $A \wedge B$, then φ is a formula A or a formula B, (3) attacks $A \rightarrow B$, then φ is a formula A, (4) defends $A \rightarrow B$, then φ is a formula B;

L2 Concession *concede* φ can be performed only by a proponent **P**, and this locution is performed when φ is an atomic formula and the performer: (1) attacks $\neg A$, then φ is a formula A, (2) defends $A \wedge B$, then φ is a formula A or a formula B, (3) attacks $A \rightarrow B$, then φ is a formula A, (4) defends $A \rightarrow B$, then φ is a formula B;

L3 Argumentation φ *since* ψ is performed when a player defends $A \vee B$, then φ is a formula $A \vee B$ and ψ is a set which includes the formula A or the formula B;

L4 Challenge The challenge *why* φ is performed when a player attacks $A \vee B$, then φ is a formula $A \vee B$;

L5 Question The question *question* φ is performed when a player attacks $A \wedge B$, then φ is a formula A or a formula B.

5.2. Protocol

In this section we propose an LND protocol describing a formal dialogue game $\triangle = m_0, m_1, \ldots, m_n$ on a topic A. We will call it DL-like game. Let **D'**(A) be DL-like dialogue for A, i.e. a set of DL-like games for A. The protocol is built upon the structural and particle rules of dialogical logic. It is specified as follows:

P1 In the first move **P** performs *claim* φ where φ is the topic A; next players perform one locution at each turn;

P2 A player **P** cannot perform *claim* φ where φ is a proposition; he can state that φ is true executing *concede* φ but this move can be performed only if **O** claimed φ in some previous move;

P3 After *claim* φ a player can perform:

1. *claim* ψ, if (a) φ is a negation of the formula and ψ is a contradiction to φ, (b) φ is the implication under the attack and ψ is the consequent of φ (**P** has to follow the restriction described in **P2**),
2. *concede* ψ, if **P** is the player and ψ is a proposition, and (a) φ is a negation of the formula and ψ is a contradiction to φ, (b) if φ is the implication under the attack and ψ is its consequent,
3. *question* ψ, if φ is a conjunction and ψ is one of its operands,
4. *why* φ, if φ is a disjunction,
5. attack or defence of any formula uttered before, if **P** is the player,
6. no move, if (a) *claim* φ is an attack on negation and φ is a proposition, (b) *claim* φ is a defence executed by **P**, and **O** has attacked this defence before;

P4 After *concede* φ performed by **P**, where φ is a proposition, **O** has:

1. no move;

P5 After φ *since* Ψ, where $\Psi = \{\psi\}$ the player can perform:

1. *claim* φ, if (a) ψ is a negation of the formula and φ is a contradiction to ψ, (b) if ψ is the implication under the attack and φ is its consequent (**P** has to follow the restriction described in **P2**),
2. *concede* φ, if **P** is the player and φ is a proposition, and (a) ψ is a negation of the formula and φ is a contradiction to ψ, (b) if ψ is the implication under the attack and φ is its consequent,
3. *question* φ, if ψ is a conjunction and φ is one of its operands,

4. *why* ψ, if ψ is a disjunction,
5. attack or defence of any formula uttered before, if **P** is the player,
6. no move, if φ *since* Ψ is a defence executed by **P**, and **O** has attacked this defence before;

P6 After *why* φ a player can perform:

1. φ *since* ψ (**P** has to follow the restriction described in **P2**),
2. attack or defence of any formula uttered before, if **P** is the player;

P7 After *question* φ a player can perform:

1. *claim* φ (**P** has to follow the restriction described in **P2**),
2. *concede* φ, if **P** is the player and φ is a proposition,
3. attack or defence of any formula uttered before, if **P** is the player;

P8 If **O** loses a game $\triangle$ which involves the *propositional choice* made by **O** (see DL-rule **SR-2**), then **O** can start a sub-game $\triangle\prime$. There are three types of sub-games $\triangle\prime$ possible:

1. Assume that **P** executes *claim* φ in $\triangle$, where φ is $\psi \wedge \psi\prime$, and **O** attacks the conjunction by stating: *question* ψ (the *propositional choice* step). If they continue to play the game $\triangle$ according to the LND rules and **P** makes the last available move, then **O** can extend $\triangle$ with a sub-game $\triangle\prime$ by attacking the conjunction one more time using the locution: *question* $\psi\prime$.
2. Assume that **O** executes *claim* φ in $\triangle$, where φ is $\psi \vee \psi\prime$. In the next moves, **P** attacks the disjunction by stating: *why* φ, and **O** defends it by stating: φ *since* ψ (the *propositional choice* step). If they continue to play the game $\triangle$ according to the LND rules and **P** makes the last available move, then **O** can extend $\triangle$ with a sub-game $\triangle\prime$ by defending the disjunction one more time with the locution: φ *since* $\psi\prime$.
3. Assume that in a game $\triangle$, **O** executes *claim* φ, where φ is $\psi \rightarrow \psi\prime$, and **P** attacks the implication by stating: *claim* ψ. There are two possible sub-cases:
   (a) Let **O** respond to this attack by defending the implication, i.e., he performs: *claim* $\psi\prime$ (the *propositional choice* step). If they continue to play the game $\triangle$ according to the LND rules and **P** makes the last available move, then **O** can extend $\triangle$ with a sub-game $\triangle\prime$ by responding to **P**'s attack one more time and attacking the propositional content of **P**'s attack, ψ, accordingly to its logical form.
   (b) Let **O** respond to **P**'s attack by attacking its content, ψ, accordingly to its logical form (the *propositional choice* step). If they continue to play the game $\triangle$ according to the LND rules and **P** makes the last available move, then **O** can extend $\triangle$ with a sub-game $\triangle\prime$ by responding to **P**'s attack one more time and defend the implication using the locution: *claim* $\psi\prime$.

In all cases **P8.1-P8.3**, during $\triangle\prime$ the players may use all the LND rules with a limitation on the **P2** rule such that **P** cannot perform *concede* ϕ if **O** did not introduce a proposition ϕ in $\triangle$ before the propositional choice step and did not introduce a proposition ϕ in $\triangle\prime$.

5.3. Effect rules

The dynamics of participants' commitments in LND formal games will be showed by a **hypothetical commitment base**. During the game, new formulas are added to this base and no formulas are deleted, since in this system the players are not allowed to retract. For a formal game $\triangle = m_0, m_1, \ldots, m_n \in$ **D'**(A), the rules for hypothetical commitment base C'_s of a player $s \in \{\mathbf{O}, \mathbf{P}\}$ are specified below, where $s(m)$ denotes a move of a player s and $\varphi, \psi \in L_t$ are propositional formulas:

E1 if $s(m_n) = claim(\varphi)$ then $C'_s(m_0, m_1, \ldots, m_n) = C'_s(m_0, m_1, \ldots, m_{n-1}) \cup \{\varphi\}$, i.e. after $claim(\varphi)$ the formula φ is added to the hypothetical commitment base,

E2 if $s(m_n) = why(\varphi)$ then $C'_s(m_0, m_1, \ldots, m_n) = C'_s(m_0, m_1, \ldots, m_{n-1})$,

E3 if $s(m_n) = concede(\varphi)$ then $C'_s(m_0, m_1, \ldots, m_n) = C'_s(m_0, m_1, \ldots, m_{n-1}) \cup \{\varphi\}$,

E4 if $s(m_n) = (\varphi \vee \psi)\ since\ \varphi$ then $C'_s(m_0, m_1, \ldots, m_n) = C'_s(m_0, m_1, \ldots, m_{n-1}) \cup \{\varphi\}$, i.e. after $(\varphi \vee \psi)\ since\ \varphi$ the formula φ is added to s's hypothetical commitment base,

E5 if $s(m_n) = question(\varphi)$ then $C'_s(m_0, m_1, \ldots, m_n) = C'_s(m_0, m_1, \ldots, m_{n-1})$.

5.4. Turntaking, termination and outcome rules

Turntaking rules: in a DL-like game (i) **P** makes the first move, then (ii) **O** and **P** take turns in performing moves.

Termination rules: a game finishes if (i) there is no legal move to perform for **O** and **P**, and (ii) **O** cannot extend the game with a sub-game.

Outcome rules: **P** wins a game if (i) the game is finished, and (ii) in the game and in all its sub-games **P** performed locution *concede*.

6. Embedding a formal dialogue in a natural dialogue

In this section we show the part of LND system which allows the embedding of formal dialogues in natural dialogues which in turn allows the representation of the motivation example as a whole.

6.1. Rules for embedding games

In this section it is specified how the protocols for natural and formal persuasion dialogues are combined. To model how the protocol for formal dialogues is embedded in the protocol for natural dialogues there is a need to introduce two new locutions: *InitLor* and *EndLor*.

L6 **Initialization** The locution *InitLor*(A) breaks the natural dialogue and initializes the DL-like dialogue for formula A, i.e., in this case *InitLor*(A) has the same meaning as *claim*(A). The player who performed *InitLor*(A) becomes the proponent for A in the embedded DL-like dialogue.

L7 Ending The locution *EndLor*(A) ends the DL-like dialogue for A and resumes the broken natural dialogue.

In our approach it is assumed that DL-like dialogue for a formula A starts when one of the players challenges this formula or states that it is not a tautology. Then, the players examine A in accordance with the rules of DL-like games. Protocol rules for embedding a formal dialogue into a natural one is described in **P9** - **P12**:

P9 The locution *InitLor*(A) can be performed as a reply to the locution: *why*(A is a tautology), or the locution: *claim*(¬ A is a tautology), executed in a natural dialogue;

P10 After the locution *InitLor*(φ) players can perform the same actions which are allowed to execute after *claim*(A) according to the dialogue rules **P1-P8**;

P11 The locution *EndLor*(A) can be performed by a player **X** if **X** has no legal move according to the dialogue rules **P1-P8**;

P12 After the locution *EndLor*(A), (1) if **P** is the performer then **P** executes *retract*(A is a tautology) in the broken natural dialogue; (2) if **O** is the performer then **O** executes *concede*(A is a tautology) in the broken natural dialogue.

6.2. Example

According to the new specification of the protocol for formal dialogues, the dialogue game examining a validity of Paul's argument can be embedded into the natural dialogue about the safety of Paul's car in the following way:

$\mathbf{P}_1$: *claim* (¬ airbag → ¬ safe)
$\mathbf{O}_2$: *concede* (¬ airbag → ¬ safe)
$\mathbf{P}_3$: *claim* (safe)
$\mathbf{O}_4$: *why* (safe)
$\mathbf{P}_5$: safe *since* {airbag; ¬ airbag → ¬ safe}
$\mathbf{O}_6$: *why* (airbag ∧ (¬ airbag → ¬ safe) → safe)
$\mathbf{P}_7$: (airbag ∧ (¬ airbag → ¬ safe) → safe) *since* {the formula $p \wedge (\neg p \to \neg q) \to q$ is a tautology}
$\mathbf{O}_8$: *claim* (¬ the formula $p \wedge (\neg p \to \neg q) \to q$ is a tautology)
 $\mathbf{P}_9$: *InitLor* $(p \wedge (\neg p \to \neg q) \to q)$
 $\mathbf{O}_{10}$: *claim* $(p \wedge (\neg p \to \neg q))$
 $\mathbf{P}_{11}$: *question* (p)
 $\mathbf{O}_{12}$: *claim* (p)
 $\mathbf{P}_{13}$: *question* $(\neg p \to \neg q)$
 $\mathbf{O}_{14}$: *claim* $(\neg p \to \neg q)$
 $\mathbf{P}_{15}$: *claim* $(\neg p)$
 $\mathbf{O}_{16}$: *claim* $(\neg q)$
 $\mathbf{P}_{17}$: *EndLor* $(p \wedge (\neg p \to \neg q) \to q)$
$\mathbf{P}_{18}$: *retract* ($p \wedge (\neg p \to \neg q) \to q$ is a tautology)

In this dialogue, the moves P_1–O_8 constitute the first part of a discussion (a natural dialogue) in which Paul makes a claim that his car is safe. The claim is obtained on the basis of reasoning (formulated in P_5) which in his opinion is valid, i.e. based on the implication that is a propositional tautology. Then, in the move P_9 Paul starts DL-like game and in moves P_9–O_{16} Paul and Olga play a formal dialogue trying to verify the implication used in Paul's reasoning. After the move O_{16}, Paul has no more legal moves available which means that he loses and in the move P_{17} ends the game. As a result, in the natural dialogue, Paul retracts from the commitment to the formal fallacy in the move P_{18}.

This example shows how natural and formal dialogues can be performed in one dialogue game. Our proposal of specification of the rules of dialogical logic enables to execute a DL-like game using it for checking the validity of reasoning. Thus, the protocol proposed in this paper offers a tool for the identification of formal fallacies (such as a fallacy of denying of the antecedent in P_5) committed during a natural dialogue.

7. The correspondence result

Dialogical logic is an example of a Game-Theoretic Semantics (GTS) [8]. A well-formed formula (WFF) is true under this semantics if and only if the proponent has a winning strategy in a specific game between proponent and opponent associated to the WFF. Therefore it is not sufficient for the proponent to win just one actual game, but he must have a winning strategy for every possible game of the associated kind between the two participants (c.f. **def. 4.1** and the structural rule **SR-3**). In this section we establish an explicit correspondence result between the dialogical logic DL and DL-like games described in this paper, i.e., that a winning strategy of a proponent **P** in DL can be turned into a winning strategy of **P** in DL-like game, and conversely. We do not have to discuss the correspondence of Prakken's dialogue system and the part of LND that describes natural dialogues, since in this paper we do not make any changes to Prakken's dialogue system. In other words, only DL is reconstructed so that it fits the style of the dialogue specification typical for natural dialogues. Let us start with defining a winning strategy in DL dialogues.

Definition 7.1. We say that a Proponent **P** has a *winning strategy* for a thesis A in DL dialogue **D**(A) if all games belonging to the dialogue **D**(A) are closed.

In DL-like dialogues a winning strategy for a proponent **P** is defined similarly.

Definition 7.2. We say that **P** has a *winning strategy* for a thesis A in DL-like dialogue **D'**(A) if in every game and sub-game belonging to **D'**(A), **P** performed locution *concede*.

The following theorem shows the correspondence between DL dialogue for a propositional formula A and DL-like dialogue for the same formula. This theorem allows for applying DL-like dialogues for verification whether a propositional formula is a tautology.

Theorem 7.1. Let A be any formula of classical propositional logic. The following conditions are equivalent:

(i) There is a winning strategy for a proponent **P** in DL dialogue **D**(A);

(ii) There is a winning strategy for a proponent **P** in DL-like dialogue **D'**(A).

Sketch of the proof. Below we present the sketch of the proof of this theorem. First we should prove that every DL-game $\triangle$ belonging to the dialogue **D**(A) has its equivalent game in DL-like dialogue **D'**(A), and vice versa. To show this it is necessary to prove the correctness of the reconstruction of the particle rules **PR-1** - **PR-4** and the structural rules **SR-0** - **SR-6**$_c$ of DL dialogues into the locution rules **L1** - **L5**, the protocol rules **P1** - **P8** and the turntaking, termination and outcome rules of DL-like dialogues. For example, consider the rules **PR-2a** and **SR-2** in DL. According to this rule, an attack on the conjunction can be made by an attack on the first or the second conjunct. This action can be realized by performing the locution *question* which has a conjunct as its propositional content. It is consistent with the locution rule **L5**. The structural rule **SR-2**, i.e., the branching rule for dialogical games, is reflected in the protocol rule **P8** which determines how branching is done in DL-like games.

On the other hand, we show that all Locution and Protocol Rules are compatible with Particle and Structural Rules. For example, consider the rule **P4**. According to this rule, after *concede* φ performed by **P**, where φ is a proposition, **O** can perform: (1) *claim* ψ, if *concede* φ is an attack on implication and ψ is its consequent; (2) no move, if (a) *concede* φ is an attack on negation and φ is a proposition, (b) *concede* φ is a defence executed by **P**, and **O** has attacked this defence before. Notice that the condition (1) is compatible with the rule **PR-4a**, i.e., an attack on negation, the condition (2a) is compatible with the rule **PR-1d** which says that there is no defence of negation, the condition (2b) is compatible with the rule **SR-5** which says that an atomic formula cannot be attacked.

The second part of the proof has to consist in showing that a winning strategy for **P** in DL means a winning strategy for **P** in DL-like. Observe that a game $\triangle$ belonging to the DL dialogue **D**(A) is closed iff there is some atomic formula which has been played in $\triangle$ by both players (c.f. **def. 7.1** and **SR-3**). In the DL-like dialogue this means that there is some atomic formula which was *claimed* by the opponent **O** first and then it was conceded by the proponent **P**. Thus, in such a game locution *concede* was performed by **P** (c.f. **def. 7.2** and the outcome rule of DL-like game). If all closed DL games belonging to **D**(A) can be reflected in all DL-like games and sub-games belonging to **D'**(A), in which **P** performed locution *concede*, then the existence of a winning strategy in DL dialogue **D**(A) for **P** is equivalent with the existence of a winning strategy in DL-like dialogue **D'**(A) for **P**.

8. Conclusions

The paper proposed a new system, called LND, for games in which the players can persuade each other not only about facts, but also about the classical propositional validity of formulas. The main contribution is the translation of dialogical logic DL from the original description to the generic specification proposed by Prakken. As a result, a DL-like formal dialogue can be easily embedded into a Prakken-like natural dialogue without a need of changing the communication language. The agents communicating about facts can shift to a dialogue allowing them to check the validity of the opponent argumentation. The system allows the players to commit a formal fallacy, but the game protocol provides a machinery for its identification by proving that the implication warranting the inference is not a propositional tautology.

Acknowledgments

We gratefully acknowledge the support of the Polish National Science Center under grant 2011/03/B/HS1/04559.

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