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Update of probabilistic beliefs: implementation and parametric verification

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Abstract. The aim of the paper is to propose how to enrich a formal model of persuasion with a specification for actions which are typical for persuasion process. First, since these actions are verbal, they influence a receiver but do not change the agent's environment. In a formal framework, we represent them as actions that change not the particular state of a model, but the whole model. Second, effects of those actions depend on how much the receiver trusts the persuader. To formally model this phenomenon, we use a trust function. Finally, we want to represent uncertainty in terms of probability. Thus far, our model did not allow to express those properties of the persuasion process. Therefore, in this paper we extend Multimodal Logic of Actions and Graded Beliefs ($\mathcal{AG}_n$) with Probabilistic Dynamic Epistemic Logic (PDEL) and elements of Reputation Management framework (RM). Incorporation of PDEL into the model of persuasion requires some modifications of

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PDEL. Such extended model is then used to enrich Perseus - our software tool that enables to examine persuasive multi-agent systems. New components of the tool allow us to execute parametric verification of the different properties related to updating probabilistic beliefs in persuasion.

1. Introduction

Persuasion is important wherever agents have to resolve conflicts and cooperate [8]. It also allows agents to influence uncertain beliefs of others [2]. Therefore, it can be effectively applied to the well-known resource re-allocation problem (RrAP) [3, 6]. This is the problem of effectively reallocating the resources such that all the agents have those resources which they need.

In the paper, we want to extend both: our formal model of persuasion and the software tool Perseus [3] such that some important properties of the persuasion process can be represented and verified in the context of RrAP applications. To this end, we use the expressivity of two logics: Multimodal Logic of Actions and Graded Beliefs ($\mathcal{AG}_n$) introduced by K. Budzynska and M. Kacprzak [1], and Probabilistic Dynamic Epistemic Logic (PDEL) proposed by B. Kooi [7]. PDEL links the probabilistically interpreted degrees of beliefs [4] and the change of epistemic states of agents [5]. However, it has some serious limitations, if we want to apply it directly to represent persuasion. Therefore, we propose an extension of syntax and semantics of PDEL. In particular, we use the elements of Reputation Management framework (RM) which allows to express the trust function (see e.g. [10, 9]).

The application of $\mathcal{AG}_n$ and PDEL to the model of persuasion for RrAP is accomplished in the following manner. First, we obtain two different uncertainty operators. One of the $\mathcal{AG}_n$ operators which is most suitable for the description of persuasion is $M!_i^{d_1,d_2}\alpha$. It intuitively means that an agent i considers d_2 doxastic alternatives (i.e. possible scenarios of a current global state) and d_1 of them satisfy α. The PDEL operator: $\mathbf{P}_i(\alpha) = q$, says that i believes α with the probability q. Both of those operators encode different and complementary properties of a persuasive system. Next, we enrich the model in order to have two types of persuasive action. The nonverbal actions are interpreted within the $\mathcal{AG}_n$ framework. The operator: $\Diamond(i : P)\alpha$, intuitively means that after executing actions P by agent i condition α may hold. In particular, α can express that an audience believes a claim with some specific degree. On the other hand, the verbal actions are interpreted similarly to public announcements in PDEL. The operator for updates: $\Diamond(i : P)\alpha$, says that α may be the case, after i performs P (i.e. after i announces something). The difference is that in the $\mathcal{AG}_n$ semantics the action P changes a state of the system, while in PDEL it changes the whole model of the system. Intuitively, nonverbal actions are physical actions and as such they change the environment of an agent (move from one state to another) and as a result they change his beliefs (the reached state may have a different accessibility relation for this agent, i.e. a relation that represents what an agent considers as possible scenarios of a current global state). On the other hand, verbal actions are "mental" actions and as such they do not change the environment of an agent, but his beliefs (they update the model). This means that the agent stays in the same state, but the model is changed so that the accessibility relation for this agent may be different. Finally, we extend the model with the use of trust function which is introduced in the framework of RM. It enables us to express which agent is perceived as credible persuader by which agent. As a result, the effect of a persuasion can be related to how much an audience trusts a given persuader. For simplicity, in this paper we consider only two trust attitudes: full trust and full distrust.

The contribution of this paper is an extension and implementation of persuasion model for RrAP with the use of two logics $\mathcal{AG}_n$ and PDEL as well as elements of RM. We add the following new components to our model: (1) an operator for probabilistic beliefs; (2) a combination of the PDEL operator with the $\mathcal{AG}_n$ operator with respect to persuasion; (3) verbal actions and their interpretation; (4) a distinction between semantics for verbal arguments and semantics of physical arguments; (5) a proposal of how to specify an update of probabilistic beliefs by means of public announcements; (6) a trust function. The new components allow us to express important properties of persuasive systems and extend our software tool Perseus [3]. As a result, the tool enables us to execute parametric verification of those properties and to apply the model in RrAP for MAS. To the best of our knowledge, there is no other tool that allows to verify the formulas with the modalities expressing updates of probabilistic beliefs induced by persuasion.

The paper is organized as follows. Section 2 introduces the basic formal notions necessary for further considerations. Section 3 shows two interpretations of uncertainty in persuasion. In Section 4, we discuss the update of uncertainty caused by verbal actions. Section 5 presents the extended $\mathcal{AG}_n$ logic. In Section 6, we show the extension of the software tool Perseus.

2. Basic formal framework

In this section, we present the starting point for our considerations. We introduce the basics of $\mathcal{AG}_n$ logic. Let V_0 be a set of *propositional variables*, Π_0 a set of *atomic actions*. Further, let ; denote a sequential composition operator. It enables to compose *schemes of programs* defined as the finite sequences of atomic *actions*: $a_1; \ldots; a_k$. Intuitively, the program $a_1; a_2$ for $a_1, a_2 \in \Pi_0$ means "Do a_1, then do a_2". The set of all schemes of programs we denote by Π.

The set of all **well-formed expressions** of $\mathcal{AG}_n$ is given by the following Backus-Naur form:

$$\alpha ::= p|\neg\alpha|\alpha \vee \alpha|M_i^d\alpha|\Diamond(i : P)\alpha,$$

where $p \in V_0$, $d \in \mathbb{N}$, $i \in Agt$.

The formulas are interpreted in a Kripke structure $\mathcal{M} = (Agt, S, RB, I, v)$ where

- $Agt = \{1, \ldots, n\}$ is a set of names of agents,
- S is a non-empty set of states (the universe of the structure),
- RB is a function which assigns to every agent a binary relation, $RB : Agt \longrightarrow 2^{S\times S}$,
- I is an interpretation of atomic actions, $I : \Pi_0 \longrightarrow (Agt \longrightarrow 2^{S\times S})$,
- v is a valuation function, $v : S \longrightarrow \{\mathbf{0}, \mathbf{1}\}^{V_0}$.

Interpretation of atomic actions I can be extended in a simple way to define interpretation of any program scheme. Let $I_\Pi : \Pi \longrightarrow (Agt \longrightarrow 2^{S\times S})$ be a function defined by mutual induction on the structure of $P \in \Pi$ as follows: $I_\Pi(a)(i) = I(a)(i)$ for $a \in \Pi_0$ and $i \in Agt$, $I_\Pi(P_1; P_2)(i) = I_\Pi(P_1)(i) \circ I_\Pi(P_2)(i) = \{(s, s') \in S \times S : \exists_{s'' \in S} \ ((s, s'') \in I_\Pi(P_1)(i) \ \text{ and } \ (s'', s') \in I_\Pi(P_2)(i))\}$ for $P_1, P_2 \in \Pi$ and $i \in Agt$.

Semantics of propositions and Boolean connectives is classical. For a given structure $\mathcal{M} = (Agt, S, RB, I, v)$ and a given state $s \in S$ the Boolean value of a formula α is denoted by $\mathcal{M}, s \models \alpha$ and is defined inductively as follows

- $\mathcal{M}, s \models p$ iff $v(s)(p) = \mathbf{1}$, for $p \in V_0$,

• $\mathcal{M}, s \models \neg\alpha$ iff $\mathcal{M}, s \not\models \alpha$,
• $\mathcal{M}, s \models \alpha \vee \beta$ iff $\mathcal{M}, s \models \alpha$ or $\mathcal{M}, s \models \beta$.

A formula $\Diamond(i : P)\alpha$ informally says that if an agent i performs a sequence of actions P then it is possible that α will be the case. Formally

• $\mathcal{M}, s \models \Diamond(i : P)\alpha$ iff $\exists_{s' \in S}$ $((s, s') \in I_\Pi(P)(i)$ and $\mathcal{M}, s' \models \alpha)$ for $P \in \Pi$.

We use also an abbreviation $\Box(i : P)\alpha$ for $\neg\Diamond(i : P)\neg\alpha$. The semantics of other formulas as well as explanation of the function RB is presented in the next sections.

3. Uncertainty in persuasion

An important aspect of persuasion is the uncertainty about a conflicting claim. To analyze this process we need to have a formal tool, with which one can express in what degree an agent believes something. Below we discuss two accounts of this issue.

3.1. Graded beliefs

So far, we exploited the formalism of graded doxastic modalities $\mathcal{AG}_n$ for reasoning about uncertainty [1]. To give some intuitions, let us consider the following example, which refers to the well-known resource re-allocation problem.

Assume there are two agents, call them John and Peter. They have access to five keys: John to the keys with identifiers 3 and 5, and Peter to the keys with identifiers 1, 2, and 4. One of the keys, in fact the key with the identifier 3, opens a safe. However, John does not know this fact, while Peter does. So the aim of Peter is to perform such an action that will give him the access to the key 3. Keys cannot be shared. Lets start at a situation in which John believes to degree $\frac{3}{5}$ that a key with an odd identifier opens the safe. Peter tries to influence John and change his degree of belief. This scenario can be represented by the model $\mathcal{M} = (Agt, S, RB, I, v)$[1] defined for the set of propositions $V_0 = \{odd\}$ and a set of atomic actions Π_0 where

- $Agt = \{John, Peter\}$ is a set of agents,
- $S = \{(S_P, S_J, S_K) : S_P, S_J, S_K \subseteq \{1, 2, 3, 4, 5\}, S_P \cap S_J = \emptyset, S_P \cup S_J = \{1, 2, 3, 4, 5\}, |S_K| = 1\}$ is a set of all states of a multi-agent system. The interpretation of a state $S = (S_P, S_J, S_K)$ is as follows. S_P is a set of Peter's keys, S_J is a set of John's keys, S_K is an one-element set containing identifier of the key which opens the safe. Sets S_P and S_J are disjoint and their union equals to the set of all number of keys,
- $v : S \longrightarrow \{\mathbf{0}, \mathbf{1}\}^{V_0}$ is a valuation function such that for every

  $s = (S_P, S_J, S_K) \in S$ it holds that $v(s)(odd) = \mathbf{1}$ iff S_K contains an odd number,
- $RB : Agt \longrightarrow 2^{S \times S}$ is a function which assigns to an agent a doxastic relation, which represents different scenarios of a current state considered by this agent. A part od its specification is given below.

[1] A function I is not needed to give the interpretation of graded beliefs. Therefore it is specified in the next sections.

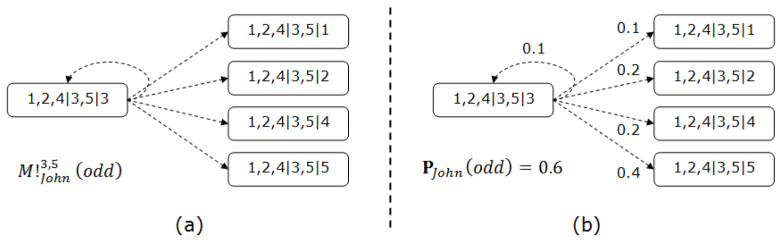

Figure 1. John's doxastic relation at the initial state in (a) the graded approach, (b) the probabilistic approach.

Let $s_0 = (\{1,2,4\},\{3,5\},\{3\})$ be the initial state of the system. For this state RB is such that $RB(Peter) = (s_0, s_0)$ and $(s_0, s) \in RB(John)$ iff $s = (S_P, S_J, S_K)$ and $S_P = \{1,2,4\}$, $S_J = \{3,5\}$, and S_K contains one of the values 1,2,3,4,5. This means that at the beginning Peter has a complete information about the actual state, while John knows what keys he has, what keys Peter has and John assumes that one of the five keys opens the safe (see Fig. 1(a)).

In $\mathcal{AG}_n$ the semantics is specified for the doxastic formula $M_i^d\alpha$. Informally it says that an agent i considers more than d doxastic alternatives of a current state in which α holds. Its formal semantics is as follows. Let $\mathcal{M}$ be a model and s a state of this model. Then, for $d \in \mathbb{N}$,

$$\mathcal{M}, s \models M_i^d\alpha \text{ iff } |\{s' \in S : (s,s') \in RB(i) \text{ and } \mathcal{M}, s' \models \alpha\}| > d.$$

Other doxastic modalities are derived from the formula $M_i^d\alpha$ in the following way:

- $B_i^d\alpha$ for $\neg M_i^d\neg\alpha$,
- $M!_i^d\alpha$ where $M!_i^0\alpha \Leftrightarrow \neg M_i^0\alpha$, $M!_i^d\alpha \Leftrightarrow M_i^{d-1}\alpha \wedge \neg M_i^d\alpha$, if $d > 0$, and
- $M!_i^{d_1,d_2}\alpha$ for $M!_i^{d_1}\alpha \wedge M!_i^{d_2}(\alpha \vee \neg\alpha)$.

Intuitively $B_i^d\alpha$ states that at most d states considered by i refute α while $M!_i^d$ means "exactly d". The most important formula that we shall use in reasoning about persuasion process is $M!_i^{d_1,d_2}\alpha$. It should be read as "i believes α with a degree $\frac{d_1}{d_2}$". Thereby, by a degree of beliefs of agents we mean the ratio of d_1 to d_2, i.e. the ratio of the number of states which are considered by an agent i and verify α to the number of all states which are considered by this agent. It is easy to observe that $0 \leq \frac{d_1}{d_2} \leq 1$. Intuitively, if an agent believes a thesis α with a degree 1 then he is absolutely sure that α holds while if he believes α with a degree 0 then he is absolutely certain α is false. Let us illustrate it on our example. John considers 5 doxastic alternatives and in three of them it is true that an odd key opens the safe. Therefore John believes to degree $\frac{3}{5}$ that the proposition odd holds. It is expressed by a doxastic formula $M!_{John}^{3,5}odd$ which is satisfied in state s_0 of the model $\mathcal{M}$. Similarly we can evaluate that John believes that an even key opens the safe to degree $\frac{2}{5}$, formally $\mathcal{M}, s_0 \models M!_{John}^{2,5}(\neg odd)$.

3.2. Probabilistic beliefs

Now assume that John, based on some information, differentiates the probabilities ("weights") of doxastic alternatives. For example, John thinks that the probability that the state, in which the key 5 opens the safe is the actual state, equals 0.4 and the probability of the states, in which the safe is opened by the keys 1 or 3, equals 0.1. Further, the probability of states, in which the keys 2 or 4 open the safe, equals 0.2. Such a situation is depicted in Fig. 1(b). Those features are not expressible in the graded modalities approach, since this framework does not allow one to distinguish states with respect to how probable they are.

To fill this gap, we add a probability function P to the model $\mathcal{M}$, which assigns to every agent i and every pair of states $(s, s') \in RB(i)$ a value from the set $[0, 1]$:

$$P : Agt \to (S \times S \to [0,1]) \text{ is a probability (partial) function such that for every agent } i \in Agt \text{ and } s \in S, \sum_{\{s' : (s,s') \in RB(i)\}} P(i)(s, s') = 1.$$

The value from the set $[0, 1]$ says how much the agent believes at state s that s' is an actual state.

The function P gives the interpretation for a probabilistic modality $\mathbf{P}$. In the extension of $\mathcal{AG}_n$ the basic probabilistic formula is $\mathbf{P}_i(\alpha) \geq q$. Informally it says that an agent i believes with probability higher or equal to q that α holds. Its semantics is as follows. Let $\mathcal{M} = (Agt, S, RB, I, P, v)$ be a model and s a state of this model. Then:

$$\mathcal{M}, s \models \mathbf{P}_i(\alpha) \geq q \quad \text{iff} \quad \sum_{\{s' \in S | (s,s') \in RB(i) \text{ and } \mathcal{M}, s' \models \alpha\}} P(i)(s, s') \geq q.$$

Formulas $\mathbf{P}_i(\alpha) = q$, $\mathbf{P}_i(\alpha) < q$, $\mathbf{P}_i(\alpha) \leq q$, $\mathbf{P}_i(\alpha) > q$ are defined from $\mathbf{P}_i(\alpha) \geq q$ in the classical way.

Let us return to our example. At the initial state John considers 3 states in which *odd* holds. The sum of all probabilities assigned for this states is $0.4 + 0.1 + 0.1 = 0.6$. So we can say that the degree of John's belief that an odd key opens the safe is 0.6. Formally, it is expressed by the probabilistic formula $\mathbf{P}_{John}(odd) = 0.6$ true at state s_0 of the model $\mathcal{M}$. Similarly, we can compute that John believes that an even key opens the safe is $0.2 + 0.2 = 0.4$, i.e. $\mathcal{M}, s_0 \models \mathbf{P}_{John}(\neg odd) = 0.4$. In this manner, we define probabilistic beliefs of agents.

3.3. Graded vs. probabilistic beliefs - comparison

Notice that on the one hand, i.e. in the graded approach, John believes *odd* to degree $\frac{3}{5}$ and on the other hand, i.e. in the probabilistic approach, he believes *odd* to degree 0.6. Obviously, $\frac{3}{5} = 0.6$. Does it mean that both formulas $M!^{3,5}_{John} odd$ and $\mathbf{P}_{John}(odd) = 0.6$ express exactly the same?

The answer depends on the information we would like to learn from these formulas. If we focus only on the degree of uncertainty then the answer is "yes", since both formulas return the same value. On the other hand, if we would like to learn something about the model in which these formulas are true then the answer is "not". Observe that $M!^{3,5}_{John} odd$ expresses that there are 5 John's doxastic alternatives and in 3 of them *odd* holds, while the formula $\mathbf{P}_{John}(odd) = 0.6$ does not describe local properties of the model with such details. So here we are dealing with a loss of the information. In other words, a probabilistic formula says what is the uncertainty of an agent about a claim, but does not give any reasons.

Losing information has serious consequences when we reason about persuasion. To show this, let us discuss two cases.

CASE 1. John assigns the probability 0.1 to a state in which key 1 opens the safe, the probability 0.5 to a state in which key 3 opens the safe, and the probability 0.2 to a state in which key 5 opens the safe. As a result, he considers 3 states in which *odd* is true and believes *odd* to probability 0.8. In this case, if Peter wants to make John believe *odd* to degree 0, then he must perform such arguments which delete doxastic options with the third component equal to $\{1\}$ or $\{3\}$ or $\{5\}$. For example, he could say "An even key opens the safe". If John accepts this argument, then Peter will achieve his goal. However, there could be such cases, in which it is difficult or even impossible to put only one successful argument forward

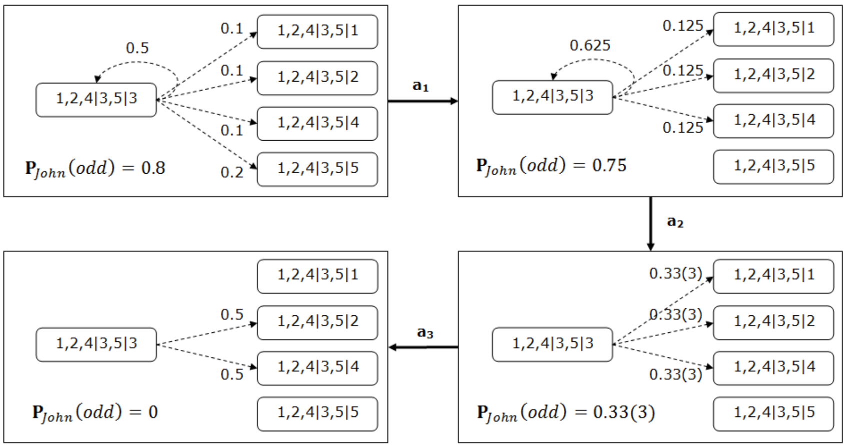

Figure 2. John's beliefs during persuasion - case 1.

which would delete all those options at once. Say that Peter has to give three arguments a_1, a_2, a_3, e.g.: "The key 5 does not open the safe", "The key 3 does not open the safe", "The key 1 does not open the safe", respectively (see Fig. 2). Every argument decreases the degree of John's belief, such that finally his uncertainty equals 0.

CASE 2. John assigns probability 0.8 to the state in which the key 5 opens the safe and probability 0.2 to the states in which the keys 2 or 4 open the safe. In consequence, he considers 1 state in which *odd* is true and believes *odd* to degree 0.8. Now, if Peter wants to make John believe *odd* to degree 0, then he must perform such arguments, which delete the option with the third component equal to $\{5\}$. Although the degree of uncertainty is the same like in the case 1, the situation is different. It may be sufficient to Peter to give only one argument, e.g. he may perform just a_1, which directly changes degree 0.8 to degree 0 (see Fig. 3).

Summing up, the main difference between graded and probabilistic modalities interpreted as beliefs is that the first one describes a situation in which persuasion starts what can help to plan a winning strategy. On the other hand, the probability formulas express only the degree of uncertainty, but in many situations it is sufficient for an evaluation of how successful arguments are. The usefulness of the above approaches strongly depends on the properties of multi-agent systems, which we would like to verify with our tool.

4. Update of uncertainty

In this section, we specify actions, which can change degrees of uncertainty during persuasion. In particular, we concentrate on verbal actions like public announcements. This type of verbal actions is taken into account in PDEL. However, we propose to enrich its interpretation by adding a trust function in order to express how much an audience trusts a persuader. Next, based on the trust function, we give a new specification of public announcement.

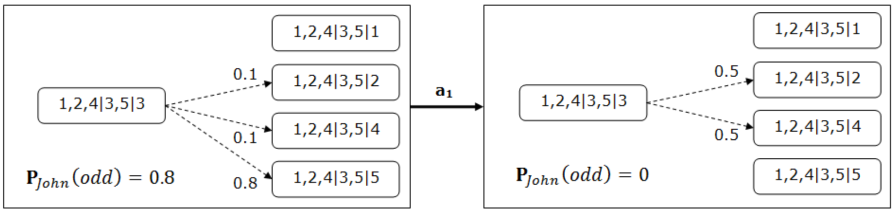

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Figure 3. John's beliefs during persuasion - case 2.

4.1. Public announcement and credibility of a proponent

A public announcement is a verbal action in which one of agents publicly declares that α holds. Formally α is a formula of a language we use for analyzing multi-agent systems. To make the model adequate to describe persuasion, we assume that every agent can hear verbal argument but his reaction depends on how much he trusts the proponent. The problem of trust is extensively studied within the framework of Reputation Management (e.g. [10, 9]). For simplicity, in this paper we consider two types of trust attitudes:

- the audience trusts the performer of a public announcement action and accepts everything he says,
- the audience does not trust the performer and disregards everything he says.

To express these attitudes in a model of a multi-agent system, we have to introduce a trust function T. In the running example T assigns to every agent i and every agent j at any state s a value from the set $\{0, 1\}$:

$$T : S \times Agt \times Agt \to \{0, 1\}.$$

Then, $T(s, i, j) = k$ for a state s, agents i, j, and $k \in \{0, 1\}$ means that i trusts j with degree k. For example, if John at the initial state s_0 trusts Peter fully and accepts everything he says, then $T(s_0, John, Peter) = 1$. If John does not trust Peter, then $T(s_0, John, Peter) = 0$.

4.2. Public announcement and verbal means

Since a public announcement is a verbal action, it does not influence the physical world in which it is uttered. Therefore, it cannot affect the valuation of propositions. In other words, the system does not move from the actual state s to a state s' such that $v(s')(p) \neq v(s)(p)$ for any proposition p. A verbal action cannot also modify the interpretation of physical actions. Moreover, for simplicity, in this specification, we disallow verbal actions to change the trust function T. What they influence is related to the cognitive states of agents, i.e. they change the doxastic relation and probability function. Furthermore, unlike PDEL, we assume that if $(s, s') \notin RB(i)$ then after the performance of a it may be the case that $(s, s') \in RB(i)$. PDEL allows only an elimination of the doxastic transitions. Thus, there is a need to define a new doxastic relation and in consequence a new model to which a system will be shifted after the performance of a verbal action. So, if an action a is performed at a state s of a model $\mathcal{M} = (Agt, S, RB, I, P, v, T)$ then after this action a system moves to the state s of an updated model $\mathcal{M}' = (Agt, S, RB', I, P', v, T)$. Observe that in the new model sets of agents, states and functions I, v, T are not changed.

Consider a public announcement action a_α, which means that a persuader says that α holds. In PDEL, an announced formula is not assumed to be true. This means that agents may update their beliefs according to α, which is a false information. A dynamic formula, which describes the result of this action is denoted by $\Diamond(i : a_\alpha)\beta$. Intuitively, it expresses that it is possible that β is true after the execution of a_α by agent i. After the performance of the action a_α an agent can modify his beliefs about α and about formulas, which describe facts connected with the fact expressed by α. However this action should not change beliefs about facts not related to α. For example, if Peter says that 3 opens the safe then John verifies his beliefs about the key, which opens the safe but does not change his beliefs about the color of the safe. Therefore, although we allow for a possibility that there exists a state s' such that $(s, s') \notin RB$ but $(s, s') \in RB'$, agents do not forget earlier beliefs but only modify some of them. This principle must be ensured by the correct definition of the doxastic relation RB, which depends on a particular application.

Again consider the running example. Let $V_0 = \{one, two, three, four, five, \ even, odd\}$ be a set of propositions where one (two, $three$, $four$, $five$, $even$, odd) means that the key 1 (2, 3, 4, 5, even, odd resp.) opens the safe. The semantics of the formula $\Diamond(i : a_p)\beta$ for $p \in V_0$ is given below. Let $\mathcal{M} = (Agt, S, RB, I, P, v, T)$ be the model and s be a state of this model. Then:

$$\mathcal{M}, s \models \Diamond(i : a_p)\beta \text{ iff } \mathcal{M}_{i,p}, s \models \beta$$

where $\mathcal{M}_{i,p} = (S, RB_{i,p}, I, P_{i,p}, v, T)$ is an updated model where $RB_{i,p}(j)$ is an updated doxastic relation and $P_{i,p}(j)(s, s')$ is an updated probability function. Let us give their definitions.

The updated doxastic relation $RB_{i,p}(j)$ is a relation such that:

- if $T(s, j, i) = 1$, $s = (S_P, S_J, S_K)$, $s' = (S'_P, S'_J, S'_K)$ then it holds that $(s, s') \in RB_{i,p}(j)$ iff $S_P = S'_P$, $S_J = S'_J$, and $\mathcal{M}, s' \models p$,
  i.e. if the agent j trusts the agent i ($i, j \in \{Peter, John\}$) then he accepts p and considers only such states in which p holds,

- if $T(s, j, i) = 0$ then it holds that $(s, s') \in RB_{i,p}(j)$ iff $(s, s') \in RB(j)$,
  i.e. if the agent j does not trust the agent i ($i, j \in \{Peter, John\}$) then he is indifferent to this agent and does not change his beliefs.

The value of the updated probabilistic function $P_{i,p}(j)(s, s')$ equals:

- $P(j)(s, s')$ if $RB_{i,p}(j) = RB(j)$,
  i.e. if the updated doxastic relation of the agent j is the same as before the public announcement action, then the probability function is also the same,

- $\frac{P(j)(s,s')}{\sum_{\{s'' \in S : (s,s'') \in RB(j) \ and \ \mathcal{M},s'' \models p\}} P(j)(s,s'')}$
  if $RB_{i,p}(j) \subset RB(j)$ and $\sum_{\{s'' \in S : (s,s'') \in RB(j) \ and \ \mathcal{M},s'' \models p\}} P(j)(s, s'') \neq 0$ and $T(s, j, i) = 1$,
  i.e. if the updated doxastic relation of the agent j is a proper subset of the doxastic relation $RB(j)$, the probability of the set of accessible states satisfying p is higher than 0, and j trusts i, then the probability function returns for every pair (s, s') the probability of (s, s') on condition that p,

- $\frac{1}{|\{s'' \in S : (s, s'') \in RB_{i,p}(j)\}|}$
  if $RB_{i,p}(j) \subset RB(j)$, $\sum_{\{s'' \in S : (s,s'') \in RB(j) \ and \ \mathcal{M},s'' \models p\}} P(j)(s, s'') = 0$ and $T(s, j, i) = 1$,

i.e. if the probability of the set of states accessible from s and satisfying p equals 0 and the agent j trusts the agent i, then the probability function returns the same probability to every pair $(s, s') \in RB_{i,p}(j)$,

- $\dfrac{1 - \sum_{\{s'' \in S : (s,s'') \in RB(j) \cap RB_{i,p}(j)\}} P(j)(s, s'')}{|\{s'' \in S : (s, s'') \in RB_{i,p}(j) \backslash RB(j)\}|}$ for $(s, s') \notin RB(j)$ and
  $P(j)(s, s')$ for $(s, s') \in RB(j) \cap RB_{i,p}(j)$, if $RB_{i,p}(j) \not\subseteq RB(j)$, $T(s, j, i) = 1$, i.e. if $RB_{i,p}(j)$ is not a subset of $RB(j)$ and the agent j trusts the agent i, then the probability function returns the old probability to all pairs (s, s') from the set $RB(j) \cap RB_{i,p}(j)$ and equal values to all new accessible states.

In general case, let Π_0^v be a set of verbal actions. Now we need to introduce a function I^v which is an interpretation for these actions. Let $\mathcal{CM}$ be a set of models and $\mathcal{CMS}$ be a set of pairs $(\mathcal{M}, s)$ where $\mathcal{M} \in \mathcal{CM}$ and s is a state of the model $\mathcal{M}$. An interpretation for verbal actions I^v is a function:

$$I^v : \Pi_0^v \longrightarrow (Agt \longrightarrow 2^{\mathcal{CMS} \times \mathcal{CMS}}).$$

Let Π^v be a set of verbal programs and $I^v_{\Pi^v}$ be an interpretation of verbal programs defined similarly to the function I_Π. The formal semantics of a formula which describes a result of an execution of a sequence of verbal actions is as follows. Let $\mathcal{M} = (Agt, S, RB, I, P, v, T)$ be a model and s a state of this model. Then

$$\mathcal{M}, s \models \Diamond(i : P)\alpha \text{ iff } \exists_{(\mathcal{M}',s') \in \mathcal{CMS}} (((\mathcal{M}, s), (\mathcal{M}', s')) \in I^v_{\Pi^v}(P)(i) \text{ and } \mathcal{M}', s' \models \alpha) \text{ for } P \in \Pi^v.$$

5. The extended $\mathcal{AG}_n$

In this section we give a formal syntax and semantics of extended $\mathcal{AG}_n$ logic, which we use as a basis for the Perseus tool. Let V_0 be a set of *propositional variables*, Π_0 a set of *atomic actions*, and Π_0^v a set of *verbal actions*. Further, let ; denote a sequential composition operator. It enables to compose *schemes of programs* defined as the finite sequences of atomic actions: $a_1; \ldots; a_k$. The set of all schemes of programs defined over Π_0 we denote by Π and the set of all schemes of verbal programs, i.e., programs defined over the set Π_0^v we denote Π^v.

The set of all **well-formed expressions** of the extended $\mathcal{AG}_n$ is given by the following Backus-Naur form:

$$\alpha ::= p|\neg\alpha|\alpha \vee \alpha|M_i^d\alpha|\Diamond(i : P)\alpha|\mathbf{P}_i(\alpha) \geq q,$$

where $p \in V_0$, $d \in \mathbb{N}$, $i \in Agt$, $q \in [0, 1]$, $P \in \Pi$ or $P \in \Pi^v$.

Other Boolean connectives are defined from $\neg$ and $\vee$ in the standard way. We use also the following abbreviations: $\Box(i : P)\alpha$ for $\neg\Diamond(i : P)\neg\alpha$, $B_i^d\alpha$ for $\neg M_i^d\neg\alpha$, $M!_i^d\alpha$ where $M!_i^0\alpha \Leftrightarrow \neg M_i^0\alpha$, $M!_i^d\alpha \Leftrightarrow M_i^{d-1}\alpha \wedge \neg M_i^d\alpha$, if $d > 0$, and $M!_i^{d_1,d_2}\alpha$ for $M!_i^{d_1}\alpha \wedge M!_i^{d_2}(\alpha \vee \neg\alpha)$. Moreover we denote $P_i(\alpha) < q$ for $\neg P_i(\alpha) \geq q$, $P_i(\alpha) = q$ for $P_i(\alpha) \geq q \wedge P_i(\alpha) \leq q$. Formulas $P_i(\alpha) \leq q$ and $P_i(\alpha) < q$ are defined in the same manner.

Definition 5.1. By a semantic model we mean a Kripke structure $\mathcal{M} = (Agt, S, RB, I, P, v, T)$ where

- $Agt = \{1, \ldots, n\}$ is a set of names of agents,
- S is a non-empty set of states (the universe of the structure),
- RB is a doxastic function which assigns to every agent a binary relation, $RB : Agt \longrightarrow 2^{S \times S}$,
- I is an interpretation of atomic actions, $I : \Pi_0 \longrightarrow (Agt \longrightarrow 2^{S \times S})$,
- P is a probability (partial) function, $P : Agt \to (S \times S \to [0, 1])$ defined for every $i \in Agt$ and $(s, s') \in RB(i)$ such that for every agent $i \in Agt$ and $s \in S$, $\sum_{\{s' : (s,s') \in RB(i)\}} P(i)(s, s') = 1$,
- v is a valuation function, $v : S \longrightarrow \{\mathbf{0}, \mathbf{1}\}^{V_0}$,
- T is a trust function, $T : S \times Agt \times Agt \to [0, 1]$.

Function I can be extended in a simple way to define interpretation of any program scheme. Let $I_\Pi : \Pi \longrightarrow (Agt \longrightarrow 2^{S \times S})$ be a function defined by mutual induction on the structure of $P \in \Pi$ as follows: $I_\Pi(a)(i) = I(a)(i)$ for $a \in \Pi_0$ and $i \in Agt$, $I_\Pi(P_1; P_2)(i) = I_\Pi(P_1)(i) \circ I_\Pi(P_2)(i) = \{(s, s') \in S \times S : \exists_{s'' \in S} \ ((s, s'') \in I_\Pi(P_1)(i) \text{ and } (s'', s') \in I_\Pi(P_2)(i))\}$ for $P_1, P_2 \in \Pi$ and $i \in Agt$.

Now we define a function I^v which is an interpretation for verbal actions.

Definition 5.2. Let $\mathcal{CM}$ be a set of models and $\mathcal{CMS}$ be a set of pairs $(\mathcal{M}, s)$ where $\mathcal{M} \in \mathcal{CM}$ and s is a state of the model $\mathcal{M}$. An interpretation for verbal actions I^v is a function:

$$I^v : \Pi_0^v \longrightarrow (Agt \longrightarrow 2^{\mathcal{CMS} \times \mathcal{CMS}}).$$

We allow different verbal actions to be executed during persuasion process. Therefore, no restrictions on I^v are assumed in the general definition. An interpretation for verbal actions will obtain different specifications depending on the type of actions and the applications of the formal model. In the paper, we describe one example of verbal actions, i.e. public announcements. Moreover, following PDEL verbal actions do not have to convey a true information. This is particularly important, if we want to use the formal framework to represent persuasion. Agents may try (successfully or not) to influence others using false messages (since they are insincere or have incomplete knowledge). Thus, we assume that I^v does not depend on the truth or falsity conditions of the announced formula. Interpretation $I^v_{\Pi^v}$ of all verbal programs is defined similarly to the function I_Π.

The **semantics** of formulas is defined with respect to a Kripke structure $\mathcal{M}$.

Definition 5.3. For a given structure $\mathcal{M} = (Agt, S, RB, I, P, v, T)$ and a given state $s \in S$ the Boolean value of the formula α is denoted by $\mathcal{M}, s \models \alpha$ and is defined inductively as follows:

$$\begin{array}{lll}
\mathcal{M}, s \models p & \text{iff} & v(s)(p) = \mathbf{1}, \ \text{for } p \in V_0, \\
\mathcal{M}, s \models \neg\alpha & \text{iff} & \mathcal{M}, s \not\models \alpha, \\
\mathcal{M}, s \models \alpha \vee \beta & \text{iff} & \mathcal{M}, s \models \alpha \text{ or } \mathcal{M}, s \models \beta, \\
\mathcal{M}, s \models M_i^d\alpha & \text{iff} & |\{s' \in S : (s,s') \in RB(i) \text{ and } \mathcal{M}, s' \models \alpha\}| > d, d \in \mathbb{N}, \\
\mathcal{M}, s \models \Diamond(i:P)\alpha & \text{iff} & \exists_{s' \in S} \ ((s,s') \in I_\Pi(P)(i) \ \text{and} \ \mathcal{M}, s' \models \alpha) \ \text{for} \ P \in \Pi \\
& & \text{or } \exists_{(\mathcal{M}',s') \in \mathcal{CMS}} \ (((\mathcal{M}, s), (\mathcal{M}', s')) \in I_{\Pi^v}^v(P)(i) \ \text{and} \\
& & \mathcal{M}', s' \models \alpha) \ \text{for} \ P \in \Pi^v, \\
\mathcal{M}, s \models \mathbf{P}_i(\alpha) \geq q & \text{iff} & \sum_{\{s' \in S | (s,s') \in RB(i) \ \text{and} \ \mathcal{M}, s' \models \alpha\}} P(i)(s,s') \geq q.
\end{array}$$

6. Perseus - implementation

The aim of the Perseus system is to analyze properties of a given semantic model $\mathcal{M} = (Agt, S, RB, I, P, v, T)$. In this case, the system input data, i.e. the **input question**, is a triple $(\mathcal{M}, s, \phi)$, where $\mathcal{M}$ is a model described by an arbitrary specification of a model (see [3]), s is a state of the model $\mathcal{M}$ and ϕ is the **input expression**. The input expression is defined by the following BNF:

$$\phi ::= \omega | \neg\phi | \phi \vee \phi | M_i^d\phi | \Diamond(i:P)\phi | M_i^?\omega | \Diamond(i:?)\,\omega | \mathbf{P}_i(\omega) \geq ? |$$

$$M_?^d\omega | \Diamond(?:P)\,\omega | \mathbf{P}_?(\omega) \geq q,$$

where $\omega ::= p | \neg\omega | \omega \vee \omega | M_i^d\omega | \Diamond(i:P)\,\omega | \mathbf{P}_i(\omega) \geq q$ and $p \in V_0, d \in \mathbb{N}$, $P \in \Pi$ or $P \in \Pi^v$, $i \in Agt$ as well as $q \in [0;1]$. Therefore the language of extended $\mathcal{AG}_n$ logic is a sublanguage of the Perseus system input expressions (what follows is that other modalities $B_i^d\omega$, $M!_i^d\omega$, $M!_i^{d_1,d_2}\omega$, $\Box(i:P)\,\omega$, $\mathbf{P}_i(\omega) > q$, $\mathbf{P}_i(\omega) = q$, $\mathbf{P}_i(\omega) < q$, $\mathbf{P}_i(\omega) \leq q$, can be derived in the standard way). Perseus system accepts two types of the input expressions:

- **unknown free expressions**, where grammar productions

  $$M_i^?\omega | \Diamond(i:?)\,\omega | \mathbf{P}_i(\omega) \geq ? | M_?^d\omega | \Diamond(?:P)\,\omega | \mathbf{P}_?(\omega) \geq q$$

  are not allowed,

- **one-unknown expression**, where only one of the grammar productions

  $$M_i^?\omega | \Diamond(i:?)\,\omega | \mathbf{P}_i(\omega) \geq ? | M_?^d\omega | \Diamond(?:P)\,\omega | \mathbf{P}_?(\omega) \geq q$$

  is allowed.

Next the Perseus system executes a **parametric verification** of an input question, i.e. tests if (both unknown free and one-unknown expressions) and when (only one-unknown expressions) the expression ϕ becomes a formula of the extended $\mathcal{AG}_n$ logic ϕ^* such that $\mathcal{M}, s \models \phi^*$. In case of unknown free expressions we have $\phi^* = \phi$, i.e a standard model verification is done. In the other case a formula ϕ^* is obtained from ϕ by swapping all ? symbols for appropriate values either from set $\{0, 1, \ldots |S|\}$ or Agt or Π or Π^v or $[0;1]$. Finally the system output data, i.e the **output answer**, is given. The output answer is *true* if $\mathcal{M}, s \models \phi^*$ and *false* otherwise. As soon as the output answer is determined, the **solution set** X for the one-unknown expression is presented, where:

- $X \subseteq \{0, 1, \ldots |S|\}$, for an expression ϕ with one unknown of type $M_i^{?}\omega$, $B_i^{?}\omega$, $M!_i^{?}\omega$, $M!_i^{?,d_2}\omega$, $M!_i^{d_1,?}\omega$,
- $X \subseteq \{0, 1, \ldots |S|\} \times \{0, 1, \ldots |S|\}$, for an expression ϕ with one unknown of type $M!_i^{?_1,?_2}\omega$,
- $X \subseteq Agt$, for an expression ϕ with one unknown of type $M_?^d\omega$, $B_?^d\omega$, $M!_?^d\omega$, $M!_?^{d_1,d_2}\omega$, $\Diamond\,(? : P)\,\omega$, $\Box\,(? : P)\,\omega$, $\mathbf{P}_?\,(\omega) \geq q$, $\mathbf{P}_?\,(\omega) > q$, $\mathbf{P}_?\,(\omega) = q$, $\mathbf{P}_?\,(\omega) < q$, $\mathbf{P}_?\,(\omega) \leq q$,
- $X \subseteq \Pi$ or $X \subseteq \Pi^v$, for an expression ϕ with one unknown of type$\Diamond\,(i :?)\,\omega$, $\Box\,(i :?)\,\omega$,
- $X \subseteq [0; 1]$, for an expression ϕ with one unknown of type $\mathbf{P}_i\,(\omega) \geq ?$, $\mathbf{P}_i\,(\omega) > ?$, $\mathbf{P}_i\,(\omega) = ?$, $\mathbf{P}_i\,(\omega) < ?$, $\mathbf{P}_i\,(\omega) \leq ?$.

In order to find an answer to the input question $(\mathcal{M}, s, \phi)$, the Perseus system executes the syntax analysis of the expression ϕ. The analysis is based on the standard descent recursive method. As a result a syntax tree of expression ϕ is created. All inner nodes of such a tree represent either Boolean operators or $\mathcal{AG}_n$ logic modalities while all outer nodes stand for either propositional variables or unknown. The solution for an arbitrary unknown is reached in the following way:

- if an unknown type is $M_i^{?}\omega$, $B_i^{?}\omega$, $M!_i^{?}\omega$, $M!_i^{?,d_2}\omega$, $M!_i^{d_1,?}\omega$, $M!_i^{?_1,?_2}\omega$, then the counting method is applied, i.e. all states, which are reachable via a doxastic relation of the agent i, and in which the thesis ω is satisfied or refuted respectively, are counted,
- if an unknown type is $M_?^d\omega$, $B_?^d\omega$, $M!_?^d\omega$, $M!_?^{d_1,d_2}\omega$, $\Diamond\,(? : P)\,\omega$, $\Box\,(? : P)\,\omega$, $\mathbf{P}_?\,(\omega) \geq q$, $\mathbf{P}_?\,(\omega) > q$, $\mathbf{P}_?\,(\omega) = q$, $\mathbf{P}_?\,(\omega) < q$, $\mathbf{P}_?\,(\omega) \leq q$, say $\mathbf{P}_?\,(\omega) \geq q$, then for every agent $i \in Agt$ the property $\mathcal{M}, s \models \mathbf{P}_i\,(\omega) \geq q$ is tested,
- if an unknown type is $\Diamond\,(i :?)\,\omega$, $\Box\,(i :?)\,\omega$, then a nondeterministic finite automaton, which represents all possible argumentation $P \in \Pi$ such that respectively $\mathcal{M}, s \models \Diamond\,(i : P)\,\omega$ or $\mathcal{M}, s \models \Box\,(i : P)\,\omega$ holds, is created,
- if an unknown type is $\mathbf{P}_i\,(\omega) \geq ?$, $\mathbf{P}_i\,(\omega) > ?$, $\mathbf{P}_i\,(\omega) = ?$, $\mathbf{P}_i\,(\omega) < ?$, $\mathbf{P}_i\,(\omega) \leq ?$, then the **summing method** is applied, i.e. probabilistic coefficients of all states, which are reachable via doxastic relation of the agent i, and in which the thesis ω is satisfied or refuted respectively, are add up.

If an unknown is a nested type, i.e. it is a part of thesis of the extended $\mathcal{AG}_n$ logic operator, then its **solution set is bounded** by the outer modality/modalities. For example, if we consider an input question

$$\left(\mathcal{M}, s, \Box\,(i : P)\, M!_j^1 \mathbf{P}_i\,(\omega) < ?\right),$$

then the solution of the unknown $\mathbf{P}_i\,(\omega) < ?$ is reduced firstly by the operator $M!$ and secondly by the operator $\Box$. The detailed description of a solution bounding problem as well as the first three types of an unknown solving methods are presented in [3]. Now, we focus only on the new unknown type, which is itemized above at the last position, i.e. probabilistic coefficient searching case. Again let us return to the running example, but consider the model given in Fig. 4(a).

Now consider the input question $(\mathcal{M}, s_0, \Diamond\,(Peter : a)\,\mathbf{P}_{John}\,(odd) \geq ?)$ with one nested unknown $\mathbf{P}_{John}\,(odd) \geq ?$. If an operator $\Diamond\,(Peter : a)$ is applied in state s_0, then the following three states are

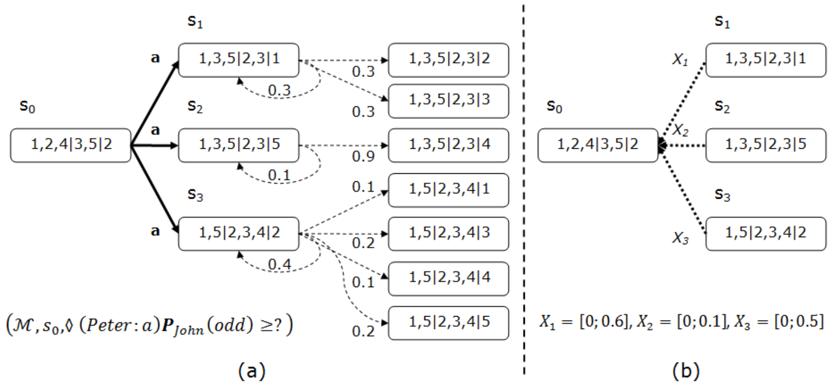

Figure 4. (a) An example of a semantic model $\mathcal{M}$. John's beliefs are marked with broken lines, the interpretation of Peter's action a is marked with bolded solid line. (b) The subsolutions X_1, X_2 and X_3 for the subquestions respectively $(\mathcal{M}, s_1, \mathbf{P}_{John}(odd) \geq ?)$, $(\mathcal{M}, s_2, \mathbf{P}_{John}(odd) \geq ?)$ and $(\mathcal{M}, s_3, \mathbf{P}_{John}(odd) \geq ?)$.

reached – s_1, s_2 as well as s_3. In every of these states the subsolution of an unknown $\mathbf{P}_{John}(odd) \geq ?$ is determined separately with the summing method. So, it stands that $\mathcal{M}, s_1 \models \mathbf{P}_{John}(odd) \geq x_1$, $\mathcal{M}, s_2 \models \mathbf{P}_{John}(odd) \geq x_2$ and $\mathcal{M}, s_3 \models \mathbf{P}_{John}(odd) \geq x_3$, where $x_1 \in X_1 = [0; 0.6]$, $x_2 \in X_2 = [0; 0.1]$ and $x_3 \in X_3 = [0; 0.5]$. The solution is finally bounded by the operator $\Diamond(Peter : a)$ (see Fig. 4(b)). In this case the answer to the question being considered is *true* and the solution for the unknown is $X_1 \cup X_2 \cup X_3 = [0; 0.6]$. Thus $\mathcal{M}, s_0 \models \Diamond(Peter : a)\mathbf{P}_{John}(odd) \geq ?$ holds for any $? \in [0; 0.6]$. The question $(\mathcal{M}, s_0, \Box(Peter : a)\mathbf{P}_{John}(odd) \geq ?)$ can be investigated in the same manner. While the subsolutions X_1, X_2 and X_3 do not change and the answer to the question is also *true*, the final solution with the outer modality $\Box(Peter : a)$ is equal to $X_1 \cap X_2 \cap X_3 = [0; 0.1]$. Therefore $\mathcal{M}, s_0 \models \Box(Peter : a)\mathbf{P}_{John}(odd) \geq ?$ holds for any $? \in [0; 0.1]$. If an input question consists of an unknown of type $\mathbf{P}_i(\omega) > ?$, $\mathbf{P}_i(\omega) = ?$, $\mathbf{P}_i(\omega) < ?$ or $\mathbf{P}_i(\omega) \leq ?$, then the solution finding method is strictly similar. For example the answer to the input question $(\mathcal{M}, s_0, \Box(Peter : a)\mathbf{P}_{John}(odd) = ?)$ is *false* because $X_1 = [0.6; 0.6]$, $X_2 = [0.1; 0.1]$, $X_3 = [0.5; 0.5]$ and $X_1 \cap X_2 \cap X_3 = \emptyset$.

7. Conclusion

In the paper, we proposed to extend the formal model of persuasion by combining the expressibility of the following frameworks: $\mathcal{AG}_n$ PDEL and RM. The uncertainty operator from PDEL allows to take into account the probability of particular scenarios (i.e. weights of states), while $\mathcal{AG}_n$ operator enables to convey information about the number of states that agent considers as possible scenarios. The action operator from PDEL allows to express the public announcement that changes only beliefs, while $\mathcal{AG}_n$ operator enables to describe the nonverbal arguments that change the physical world. Moreover, we introduce the new component of trust function. The parametric verification of these new components is possible by means of the Perseus tool.

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