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Perseus. Software for Analyzing Persuasion Process

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Abstract. The aim of the paper is to present the software tool Perseus and show how it can be used to examine multi-agent systems where the ability to persuade is specified. Especially we want to study the issues such as: what arguments individuals use to successfully convince others, what type of a persuader guarantees a victory etc. This work describes implementation of the tool and discusses what questions about persuasion process Perseus can answer and how it is done.

1. Introduction

A great deal of interest has focused recently on the argumentation and persuasion. The obvious consequence was the question if there is possibility of implementing the theories of these processes. Last

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years have brought several proposals of **software systems for argumentation**. ANA (Automated Negotiation Agent) uses a logical model of mental states of agents based on a representation of their beliefs, desires, intentions and goals. The model specifies argument formulation and evaluation. The user of ANA can analyze and explore different methods to negotiate and argue [12]. CaSAPI is an argumentation system implemented in Prolog that combines abstract and assumption-based argumentation [5]. Another proposal is an implementation of DeLP which combines Defeasible Argumentation and Logic Programming. The application is a multi-agent system for the stock market domain. Agents reason using DeLP and are capable of formulating arguments and counterarguments in order to decide whether to buy or sell some stock [6]. Araucaria is a software tool for graphical argument representation and analysis [17]. ArgDF is a Semantic Web-based system, i.e., an implementation of the Argument Interchange Format (AIF) in a Semantic Web language. ArgDF is an argumentation support system with decentralized construction distributed across contributors and software developers in the model of Web 2.0 applications [16]. Argument-assistance systems (such as Argue! or ArguMed) are aids to draft and generate arguments, e.g. by supervising the argument process or checking if the system users obey the rules of argument [20]. The Athena software consists of two software packages along with educational modules. One package is designed to support reasoning and argumentation and another - to facilitate analysis of decisions and two-party negotiations [18]. The Reason! Project is a software for improving informal reasoning skills [7]. Carneades is an Open Source software library for building tools supporting a variety of argumentation tasks [8].

The **novelty** of our work is the **research subject** and its **aim**. That is, Perseus does not implement any theory of argumentation, but the theory of **persuasion** and - in addition - persuasion understood in a specific rhetorical manner. This needs some explanation. We base the notion of persuasion on the definition introduced by D. Walton and E. Krabbe [21]. Persuasion dialogue is a dialogue of which initial situation is a conflict of opinion and the aim is to resolve this conflict and thereby influence the *change of agents' beliefs*. Following such the definition, two approaches of studying the persuasion process are possible. One approach concentrates on the conflict of opinion and is called dialectical, while the other one focuses on the influence on beliefs and is called *rhetorical*. The dialectical research deals with the issues concentrated around the question "How to persuade to resolve the conflict?". In consequence, these theories investigate the problems such as: designing the protocols which allow reaching the resolution of the conflict, providing a list of speech acts which are viewed as moves regulated by the persuasion game's rules (protocol) or handling the inconsistent belief bases of agents by means of argumentation based formalizations. Our theory is the example of the rhetorical approach. Thus our framework deals with the issues concentrated around the question "What impact on beliefs has a particular persuasion?". In consequence, we investigate the problems such as: the degree and the scenario of belief changes, the factors that influence them, the strength of different types of arguments and their arrangements, the credibility of persuader, etc. Moreover, we assume the broader notion of persuasion than Walton and Krabbe. That is, our framework allows such acts of convincing that are not dialogues what enables to represent and analyze nonverbal arguments.

The other novelty of our work is the aim of the studies. We do not want to design the persuasion system in the analogues manner to the argumentation systems described above. Instead, our aim is to design the system which allows to **investigate such persuasion systems**. In consequence, we do not intend to build new models of persuasion, develop and implement arguing agents, determine their architecture and specification or define protocols of dialogue games. We rather look for adequate (not only new but also existing) solutions and tools to study the process of persuasion and its selected properties such as: effects, success, strength, dynamics, etc. Therefore, we want to have - on the one hand - a logic which allows to express those properties of persuasion systems and on the other hand - a software system which allows to examine selected multi-agent systems with respect to those properties. In the papers [1, 2] we introduced the formal model of multi-agent systems and a modal logic $\mathcal{AG}_n$ which language is interpreted in this model. On this base we are able to syntactically (deductively) test validity of formulas which express specification of arguing agents as well as properties of systems that can be expressed via our formal model. In this paper, we show a **software system** Perseus. It offers two main options of investigation. First, it can semantically *verify satisfaction of formulas* of the $\mathcal{AG}_n$ language which describe properties under consideration in a given model. Second, it can *search for answers to questions* of three kinds - questions about the degrees of uncertainty, questions about the sequence of arguments that should be executed and questions about the agents participating in the process of persuasion.

The paper is organized as follows. Section 2 shows a logical framework on which Perseus is based. Section 3 gives the main ideas of the tool. Section 4 presents an example on the basis of which we show application of Perseus. Section 5 describes input data, i.e. focuses on specification and implementation of a model of a multi-agent system as well as formulation of questions. Section 6 discusses methods explored in finding answers for questions. Section 7 presents concluding remarks.

2. $\mathcal{AG}_n$ logic

Perseus is a tool for analyzing persuasion process. Main aspects of this process we want to study are: uncertainty of an agent about a given thesis and change of this uncertainty caused by specific argumentation. Therefore our tool is based on $\mathcal{AG}_n$ logic [1], which allows for expressing both of these aspects. In order to reason about uncertainty we use the formula $M!_i^{d_1,d_2}\alpha$ which says that an agent i considers d_2 doxastic alternatives (i.e. visions of a current global state) and d_1 of them satisfy the condition α. Intuitively it means that the agent i believes with degree $\frac{d_1}{d_2}$ that α holds. We can also express that more than d doxastic alternatives of the agent i satisfy α (without pointing how many alternatives the agent considers): $M_i^d\alpha$ or that at most d doxastic alternatives refute α: $B_i^d\alpha$. In our approach argumentation is treated as a sequence of actions. Thereby for reasoning about the change of uncertainty caused by specific argumentation the formula $\Diamond(i : P)\alpha$ is used. It says that after giving argumentation P by agent i the condition α may hold. Now we shortly describe formal syntax and semantics of $\mathcal{AG}_n$. For more details see [1, 2].

Let $Agt = \{1, \ldots, n\}$ be a set of names of *agents*, V_0 be a set of *propositional variables*, and Π_0 a set of *program variables*. Further, let ; denote a programme connective which is a sequential composition operator. It enables to compose *schemes of programs* defined as the finite sequences of atomic **actions**: $a_1; \ldots; a_k$. Intuitively, the program $a_1; a_2$ for $a_1, a_2 \in \Pi_0$ means "Do a_1, then do a_2". The set of all schemes of programs we denote by Π.

The set of all **well-formed expressions** of $\mathcal{AG}_n$ is given by the following Backus-Naur form (BNF):

$$\alpha ::= p|\neg\alpha|\alpha \vee \alpha|M_i^d\alpha|\Diamond(i : P)\alpha,$$

where $p \in V_0$, $d \in \mathbb{N}$, $P \in \Pi$, $i \in Agt$. Other Boolean connectives are defined from $\neg$ and $\vee$ in the standard way. We use also the following abbreviations: $\Box(i : P)\alpha$ for $\neg\Diamond(i : P)\neg\alpha$, $B_i^d\alpha$ for $\neg M_i^d\neg\alpha$, $M!_i^d\alpha$ where $M!_i^0\alpha \Leftrightarrow \neg M_i^0\alpha$, $M!_i^d\alpha \Leftrightarrow M_i^{d-1}\alpha \wedge \neg M_i^d\alpha$, if $d > 0$, and $M!_i^{d_1,d_2}\alpha$ for $M!_i^{d_1}\alpha \wedge M!_i^{d_2}(\alpha \vee \neg\alpha)$.

Definition 2.1. Let Agt be a finite set of names of agents. By a semantic model we mean a Kripke structure $\mathcal{M} = (S, RB, I, v)$ where

- S is a non-empty set of states (the universe of the structure),
- RB is a doxastic function which assigns to every agent a binary relation, $RB : Agt \longrightarrow 2^{S\times S}$,
- I is an interpretation of the program variables, $I : \Pi_0 \longrightarrow (Agt \longrightarrow 2^{S\times S})$,
- v is a valuation function, $v : S \longrightarrow \{\mathbf{0}, \mathbf{1}\}^{V_0}$.

Function I can be extended in a simple way to define interpretation of any program scheme. Let $I_\Pi : \Pi \longrightarrow (Agt \longrightarrow 2^{S\times S})$ be a function defined by mutual induction on the structure of $P \in \Pi$ as follows: $I_\Pi(a)(i) = I(a)(i)$ for $a \in \Pi_0$ and $i \in Agt$, $I_\Pi(P_1; P_2)(i) = I_\Pi(P_1)(i) \circ I_\Pi(P_2)(i) = \{(s, s') \in S \times S : \exists_{s''\in S} \quad ((s, s'') \in I_\Pi(P_1)(i) \quad \text{and} \quad (s'', s') \in I_\Pi(P_2)(i))\}$ for $P_1, P_2 \in \Pi$ and $i \in Agt$.

The **semantics** of formulas is defined with respect to a Kripke structure $\mathcal{M}$.

Definition 2.2. For a given structure $\mathcal{M} = (S, RB, I, v)$ and a given state $s \in S$ the Boolean value of the formula α is denoted by $\mathcal{M}, s \models \alpha$ and is defined inductively as follows:

$$\begin{array}{lll}
\mathcal{M}, s \models p & \text{iff} & v(s)(p) = \mathbf{1}, \ \text{ for } \ p \in V_0, \\
\mathcal{M}, s \models \neg\alpha & \text{iff} & \mathcal{M}, s \not\models \alpha, \\
\mathcal{M}, s \models \alpha \vee \beta & \text{iff} & \mathcal{M}, s \models \alpha \text{ or } \ \mathcal{M}, s \models \beta, \\
\mathcal{M}, s \models M_i^d \alpha & \text{iff} & |\{s' \in S : (s, s') \in RB(i) \text{ and } \ \mathcal{M}, s' \models \alpha\}| > d, \, d \in \mathbb{N}, \\
\mathcal{M}, s \models \Diamond(i : P)\alpha & \text{iff} & \exists_{s'\in S} \ ((s, s') \in I_\Pi(P)(i) \ \text{ and } \ \mathcal{M}, s' \models \alpha).
\end{array}$$

We say that α is true in a model $\mathcal{M}$ at a state s if $\mathcal{M}, s \models \alpha$.

3. Perseus system

The aim of the Perseus system is to analyze properties of any multi-agent system, which can be well-described using the $\mathcal{AG}_n$ formalism. In this case the system input data, i.e. input question, is compounded with three parts. The first one is a specification of a model of a multi-agent system. The second one is an arbitrary state of a model of a multi-agent system. The last one is an expression, which represents a property of a multi-agent system. Next, the Perseus system executes a **parametric verification** of an input question, i.e. tests if and when a property of a multi-agent system is true. The output data is an answer to a question, which can be either $true$ or $false$. If a positive answer is given, then a solution, i.e. criteria when an expression is true, are presented – see Fig. 1.

Along this approach a multi-agent system expert is designated to build a specification of a model of a multi-agent system (this task is not a part of this research). Finally, an user of the Perseus system gives an input question and then learns about properties of the multi-agent system. For example “What will a degree of an agent’s belief in some thesis be after a specific argumentation is done?”, “Which argumentation should be executed to change a degree of an agent’s belief in some thesis to a specified level?”.

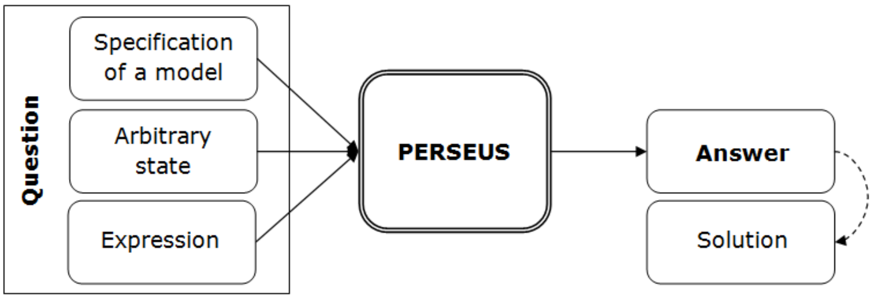

Figure 1. The idea of Perseus computer system.

4. The resource re-allocation problem

In this section we present an application of Perseus system in the field of the resource re-allocation problem. This problem can be intuitively described as the process of re-distributing a number of items (resources) amongst a number of agents. In our approach the re-distributing process is strongly related with the agents' persuasion. The instances of the resource re-allocation problem, where the resources are divisible or not, can be found in many **practical applications**, for example, in the logistic (the airport traffic management [11], public transport [3]), in the informatics technology (network routing [4], grid architectures and computations [9]), in the industry and commerce (industrial procurement and scheduling [15, 19]), but also in the management of the Earth Observation Satellites project [14, 13].

The resource re-allocation problem is also discussed in [10]. The focus of this paper is on information-seeking and **negotiations** which can provide a solution to this problem. In the described approach, agents begin with beliefs specifying which resources they have and which resources they would like to have. Then, they communicate in order to establish which agent has the desired resources and when it is fixed they start negotiations. A negotiation dialogue, as defined in the work, allows agents to agree or disagree on an exchange of these resources. More specifically, if an agent wishes to exchange a resource with another agent, it sends an offer indicating the resources to be given and received in the exchange. The agent receiving the offer can either accept or reject it. In this manner the paper enriches the resource re-allocation problem with the aspect of negotiation, however the applied one-to-one negotiations are assumed to be very simple. In our work, we would like to enrich these negotiations with persuasion. Therefore the accompanied argumentation process is also very trivial. We extend this approach by allowing agents to **persuade** each other and give diverse arguments in order to achieve desired resources. In consequence, we can investigate richer and complex argumentation, which can be successfully and automatically analyzed with our tool Perseus. In order to illustrate how the Perseus works, consider the following example of the resource re-allocation problem. We will relate to it in the next sections.

Example. Assume a system with two agents: John and Ann. Both agents are aware that in the world they exist there are five keys, two of which are needed to open a safe. The states of the system are specified by three components and doxastic relations assigned to them. The first component lists numbers of keys belonging to Ann, the second one lists numbers of keys belonging to John, and the third one indicates which keys open the safe. It is possible that two states have the same components but different doxastic relations. At the beginning Ann has keys number one, two, and four while John has keys three and five. The keys which open the safe are also three and five. So, the initial state s_1 of the system is specified by $(1, 2, 4|3, 5|3, 5)$. In this state John knows which keys he has. He also knows that Ann has the other keys, but does not know which keys open the safe. Therefore he allows for a possibility all combinations of two keys. That is, he considers as his doxastic alternatives all states in the form $(1, 2, 4|3, 5|x, y)$, where x, y are possible combinations of keys opening the safe. The doxastic relation of John assigned to state s_1 is depicted in Fig. 2. Furthermore, we assume only one

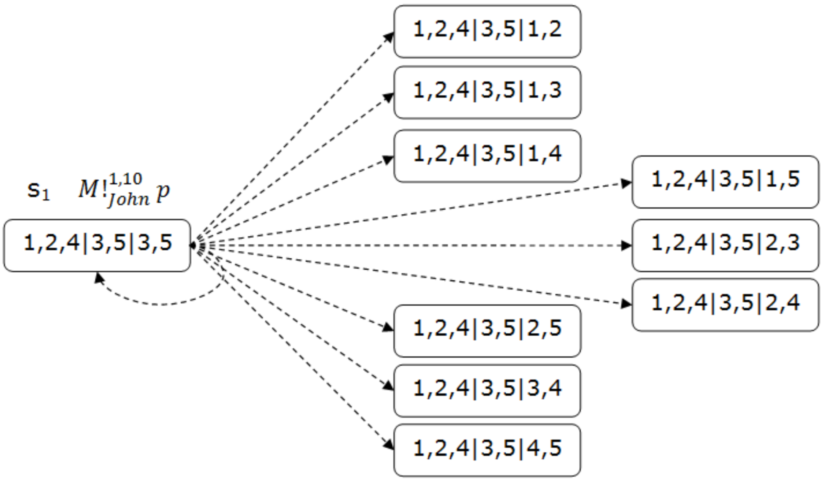

Figure 2. Doxastic relation of John in the state s_1.

proposition p expressing that "John has the keys which open the safe". Observe that p is satisfied only in one John's doxastic alternative. In consequence it holds that $s_1 \models M!^{1,10}_{John}p$ - John believes with the degree $\frac{1}{10}$ that he has desired keys. The goal of Ann's activity is to wheedle keys opening the safe from John by exchanging the keys. In order to do this, she will try to convince John that such an exchange will be profitable for him, i.e. after the exchange John will believe p with higher degree (or $\neg p$ with lower degree). All actions which Ann performs can change components of a state as well as doxastic alternatives of both agents.

5. Question, answer and solution

In this section we show how a model of an input multi-agent system is implemented and how initial questions are formulated. The research is followed by an assumption that we are given a $\mathcal{AG}_n$ logic compatible specification of a model of a multi-agent system. The **specification of a model** is a full characteristic of a semantic model $\mathcal{M} = (S, RB, I, v)$. On this ground Perseus system automatically generates the **implementation of a model**, i.e. an object which is consisted of the following attributes and one method:

- `var`$[1..|V_0|]$ – vector of unique names of propositional variables,
- `st`$[1..|S|]$ – vector of unique names of states,

- $\mathtt{agt}[1..|Agt|]$ – vector of unique names of agents,
- $\mathtt{act}[1..|\mathit{\Pi}_0|]$ – vector of unique names of actions,
- T – the method of name translation $T(\mathtt{vector}, n) = x$, where
  - $\mathtt{vector}$ is a vector $\mathtt{var}$, $\mathtt{st}$, $\mathtt{agt}$ or $\mathtt{act}$,
  - n is an element of the set V_0, S, Agt or Π_0 respectively,

  such, that x is an index of an element of the vector $\mathtt{vector}$, for which $\mathtt{vector}[x]$ represents element n,
- $\mathtt{value}[1..|S|]$ – valuation function vector, where $\mathtt{value}[\alpha(\mathtt{st}, s)]$ is a Boolean vector of length $|V_0|$, which represents value $v(s)$, for $s \in S$,
- $\mathtt{doxastic}[1..|Agt|]$ – doxastic function vector, where $\mathtt{doxastic}[T(\mathtt{agt}, i)]$ is a square matrix of rank $|S|$, which represents relation $RB(i)$, for $i \in Agt$,
- $\mathtt{interpret}[1..|\mathit{\Pi}_0|]$ – action interpretation vector, where $\mathtt{interpret}[T(\mathtt{act}, a)]$ is a vector of length $|Agt|$ such that $(\mathtt{interpret}[T(\mathtt{act}, a)])[T(\mathtt{agt}, i)]$ is a square matrix of rank $|S|$, which represents relation $(I(a))(i)$, for $a \in \Pi_0$ and $i \in Agt$.

As soon as an implementation of a model is generated, the properties of this model can be analyzed. In order to do that an input question is introduced. Now we present how such a question can be constructed and which types of questions we consider.

The **input question** to the Perseus system is a triple

$$(\mathcal{M}, s, \phi),$$

where $\mathcal{M}$ is a model described by an arbitrary specification of a model, s is a state of this model and ϕ is the **input expression** defined by the following BNF form:

$$\phi ::= \omega|\neg\phi|\phi \vee \phi|M_i^d\phi|\Diamond(i : P)\phi|M_i^?\omega|\Diamond(i :?)\,\omega, |M_?^d\omega|\Diamond(? : P)\,\omega,$$

where

$$\omega ::= p|\neg\omega|\omega \vee \omega|M_i^d\omega|\Diamond(i : P)\,\omega$$

and $p \in V_0$, $d \in \mathbb{N}$, $P \in \Pi$, $i \in Agt$. Therefore, the language of $\mathcal{AG}_n$ logic is a sublanguage of the language of the Perseus system input expressions. It follows that, the previously presented modalities B_i^d, $M!_i^d$, $M!_i^{d_1,d_2}$ and $\Box(i : P)$ can be derived in the same way.

In this paper we focus only on two types of the input expressions. The expression of the first type is the **unknown-free expression** and it is obtained if grammar productions $M_i^?\omega|\Diamond(i :?)\,\omega, |M_?^d\omega|\Diamond(? : P)\,\omega$ are not used to derive ϕ. For example:

- $M!_i^3\omega$, exactly 3 doxastic alternatives of the agent i satisfy the thesis ω,
- $M_i^4 B_j^2\omega$, in more than 4 doxastic alternatives of the agent i it is true that in at most 2 doxastic possibilities of the agent j the thesis ω is refuted,

- $\Diamond\,(i:P)\,\omega$, the execution of the argumentation P by the agent i may cause that the thesis ω is satisfied,
- $\Box\,(i:P)\,M!_j^{2,4}\omega$, the execution of the argumentation P by the agent i can not cause that it is not true that in exactly 2 doxastic alternatives of the agent j among exactly 4 his doxastic possibilities the thesis ω is satisfied.

The expression of the second type is the **one-unknown expression** and it is constructed if grammar productions $M_i^?\omega|\Diamond\,(i:?)\,\omega, |M_?^d\omega|\Diamond\,(?:P)\,\omega$ are used to derive ϕ, where expression ϕ consists of only one **unknown** from the list of possible unknowns, for example:

- $M_i^?\omega$, in more than how many doxastic alternatives of the agent i the thesis ω is satisfied?
- $M_?^d\omega$, for which agent is it true that in more than d of his doxastic alternatives the thesis ω is satisfied?
- $M!_i^{?,d_2}\omega$, in exactly how many doxastic alternatives of the agent i, from exactly d_2 of his doxastic possibilities, the thesis ω is satisfied?
- $M!_i^{d_1,?}\omega$, what is an exact number of all doxastic alternatives of the agent i, where in exactly d_1 of them the thesis ω is satisfied?
- $M!_i^{?_1,?_2}\omega$, what is an exact number of all doxastic alternatives of the agent i and in exactly how many of them the thesis ω is satisfied?
- $M!_?^{d_1,d_2}\omega$, for which agent is it true that in exactly d_1 doxastic alternatives among exactly d_2 of his doxastic possibilities the thesis ω is satisfied?
- $\Diamond\,(i:?)\,\omega$, for what argumentation is it true that its execution by the agent i may cause that the thesis ω is satisfied?
- $\Diamond\,(?:P)\,\omega$, for which agent is it true that his execution of the argumentation P may cause the thesis ω is satisfied?

The **answer** to an input question $(\mathcal{M}, s, \phi)$ is $true$, if there exists a formula ϕ^* of the language of the $\mathcal{AG}_n$ logic such that $\mathcal{M}, s \models \phi^*$ holds. A formula ϕ^* is obtained from an expression ϕ by swapping all ? symbols into appropriate values either from the set $\{0, 1, \ldots, |S|\}$ or Agt or Π. In other case, i.e. a formula ϕ^* does not exists, the answer is $false$.

The set X of all possible values of ? symbols, for which the answer to an input question $(\mathcal{M}, s, \phi)$ is $true$, is called the **solution of an unknown** and:

- $X \subseteq \{0, 1, \ldots, |S|\}$, for an expression ϕ with one of the unknowns $M_i^?\omega$, $B_i^?\omega$, $M!_i^?\omega$, $M!_i^{?,d_2}\omega$, $M!_i^{d_1,?}\omega$,
- $X \subseteq \{0, 1, \ldots, |S|\} \times \{0, 1, \ldots, |S|\}$, for an expression ϕ with an unknown $M!_i^{?_1,?_2}\omega$,
- $X \subseteq Agt$, for an expression ϕ with one of the unknowns $M_?^d\omega$, $B_?^d\omega$, $M!_?^d\omega$, $M!_?^{d_1,d_2}\omega$, $\Diamond(?:P)\omega$, $\Box(?:P)\omega$,

- $X \subseteq \Pi$, for an expression ϕ with one of the unknowns $\Diamond(i:?)\omega$, $\Box(i:?)\omega$.

If an input expression ϕ is unknown-free one, then it is simply a formula of the language of the $\mathcal{AG}_n$ logic. Thus, the answer is $true$ if and only if $\mathcal{M}, s \models \phi$ and no solution set is considered.

6. Answering questions

In this section we describe how the Perseus system finds an answer to an input question $(\mathcal{M}, s, \phi)$. This process is divided into two stages. At the beginning a syntax analysis of a thesis ϕ is applied and then optionally at most one unknown is considered. We briefly study three possible cases – coefficient's searching case, agent's searching case and the most complex argumentation's searching case.

The syntax analysis of a thesis ϕ is based on a standard descent recursive parser method, which uses the BNF grammar of an expression ϕ presented in the previous section. As a result a syntax tree for an expression ϕ is created. All inner nodes of such a tree represent either standard Boolean operators or $\mathcal{AG}_n$ logic modalities. The outer nodes stand for either propositional variables or unknown. Still, at most one unknown can be reached and if so it is, a solution is determined in the following way:

- if an unknown type is $M_i^?\omega$, $B_i^?\omega$, $M!_i^?\omega$, $M!_i^{?,d_2}\omega$, $M!_i^{d_1,?}\omega$, $M!_i^{?_1,?_2}\omega$, then the **counting method** is applied, i.e. all states, which are reachable via doxastic relation of the agent i, and in which the thesis ω is satisfied or refuted respectively, are counted,
- if an unknown type is $M_?^d\omega$, $B_?^d\omega$, $M!_?^d\omega$, $M!_?^{d_1,d_2}\omega$, $\Diamond\,(?:P)\,\omega$, $\Box\,(?:P)\,\omega$, say $M_?^d\omega$, then for every agent $i \in Agt$ the property $\mathcal{M}, s \models M_i^d\omega$ is tested,
- if an unknown type is $\Diamond\,(i:?)\,\omega$, $\Box\,(i:?)\,\omega$, then a nondeterministic finite automaton, which represents all possible argumentation $P \in \Pi$ such that respectively $\mathcal{M}, s \models \Diamond\,(i:P)\,\omega$ or $\mathcal{M}, s \models \Box\,(i:P)\,\omega$ holds, is created.

If an unknown is a part of a thesis of a $\mathcal{AG}_n$ logic operator, then its **solution set is bounded** by the proper outer modality. For example, if we consider an input question $\left(\mathcal{M}, s, \Box\,(i:P)\,M!_j^1 B_i^?\omega\right)$, then solution of the unknown $B_i^?\omega$ is reduced firstly by the operator $M!$ and secondly by the operator $\Box$. More detailed description of methods of unknown solving and examples of solution bounding are studied in subsections 6.1 and 6.2.

6.1. Coefficient's or agent's searching case

Let's consider again the example of a simple multi-agent system introduced in Section 4. Now we assume that in the state s_1 Ann is able to execute the action a_1. Due to action's specification Ann offers to John an exchange of key two for key three. She justifies an action's necessity with a statement, which is obviously false, that in order to open the safe one odd and one even key is necessary. The result of action a_1 is strongly determined by John's attitude. Now we want to track the change of degree of John's belief about the thesis $\neg p$ (it is not true that John has the correct set of keys, which enables him to open the safe) after the action a_1 is executed by Ann - see Fig. 3(a).

If John trusts Ann, then he agrees to the keys' exchange and what is more he begins to consider only these doxastic alternatives in which the safe may be opened only with the pair of odd/even keys. Thus

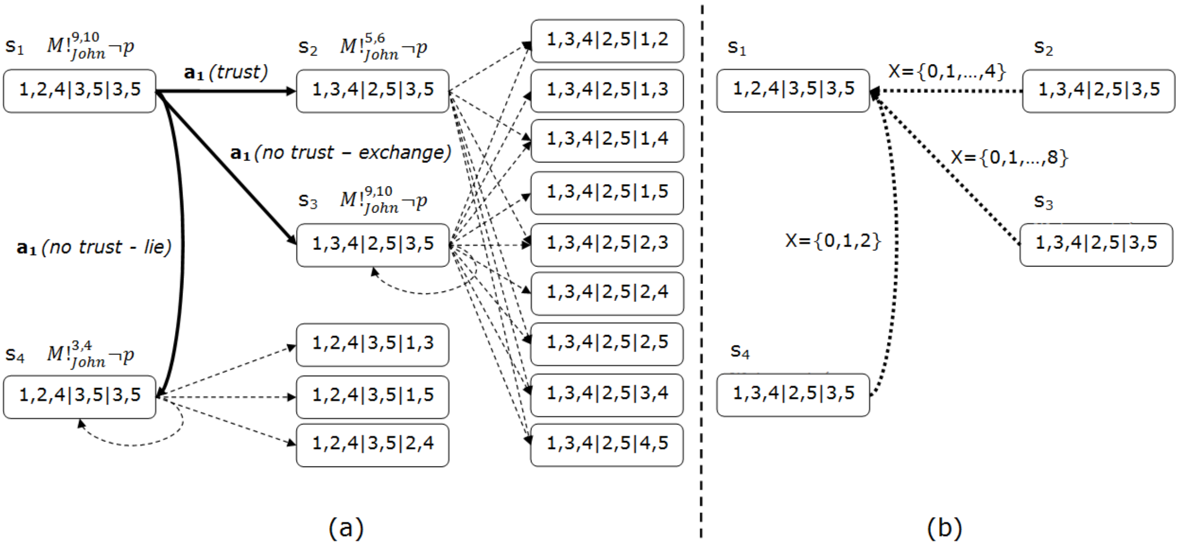

Figure 3. (a) An example model $\mathcal{M}$. Doxastic relation of John is marked with broken lines, results of action a_1 done by Ann in state s_1 are marked with bolded solid lines. (b) The solutions for the subquestions $(\mathcal{M}, s_2, M^?_{John}\neg p)$, $(\mathcal{M}, s_3, M^?_{John}\neg p)$ and $(\mathcal{M}, s_4, M^?_{John}\neg p)$.

the result of the action a_1 is the state $s_2 = (1,3,4|2,5|3,5)$ such that $s_2 \models M!^{5,6}_{John}\neg p$. If John does not trust Ann, then he can respond to the action a_1 in two different ways. The first one is that John agrees to the keys' exchange but he doesn't reset his beliefs. In this case the state s_3 is achieved, where $s_3 \models M!^{9,10}_{John}\neg p$. The second way determines that John assumes that Ann is lying. Therefore, John does not accept the exchange and begins to consider only these doxastic alternatives in which the safe may be opened only with the pair of odd/odd or even/even keys. This kind of John's reaction is described by the state s_4 such that $s_4 \models M!^{3,4}_{John}\neg p$.

Now we consider the input question $(\mathcal{M}, s_1, \Diamond(Ann : a_1)\, M^?_{John}\neg p)$, where $\mathcal{M}$ is an example model being discussed. If the operator $\Diamond(Ann : a_1)$ is applied in the state s_1 of the model $\mathcal{M}$, then the following states are reached – s_2, s_3, s_4. In every of these states the unknown $M^?_{John}\neg p$ is investigated separately and its solution is found by the counting method. It follows that $\mathcal{M}, s_2 \models M^{x_{s_2}}_{John}\neg p$, $\mathcal{M}, s_3 \models M^{x_{s_3}}_{John}\neg p$ and $\mathcal{M}, s_4 \models M^{x_{s_4}}_{John}\neg p$ holds, where $x_{s_2} \in \{0,1,...,4\}$, $x_{s_3} \in \{0,1,...,8\}$ and $x_{s_4} \in \{0,1,2\}$. This subsolutions are finally bounded by the operator $\Diamond(Ann : a_1)$ – see Fig. 3(b). In this case the final solution for the nested unknown $\Diamond(Ann : a_1)\, M^?_{John}\neg p$ is $X = \{0,1,...,4\} \cup \{0,1,...,8\} \cup \{0,1,2\}$. Therefore, the answer to the input question is $true$ and $? \in \{0,1,...,8\}$. The question $(\mathcal{M}, s_1, \Box(Ann : a_1)\, M^?_{John}\neg p)$ can be investigated similarly. While the subsolutions does not change, the final solution for the nested unknown $\Box(Ann : a_1)\, M^?_{John}\neg p$ is $X = \{0,1,...,4\} \cap \{0,1,...,8\} \cap \{0,1,2\}$, that is the answer is $true$ if and only if $? \in \{0,1,2\}$.

If an input question consists of an unknown of the type $M^d_?\omega$, $B^d_?\omega$, $M!^d_?\omega$, $M!^{d_1,d_2}_?\omega$, $\Diamond(? : P)\,\omega$, $\Box(? : P)\,\omega$, then a solution finding method is strictly similar. The difference is that the subresults are determined by testing an unknown for all possible valuations of symbol ?, i.e. $? = i$, where $i \in Agt$. Thus, $? \in X$ for some $X \subseteq Agt$.

6.2. Argumentation's searching case

Let's reconsider the example model $\mathcal{M}$. In order to present our approach in the case of finding solution for the program unknown, i.e. $\Diamond (i :?)\, \omega$, $\Box (i :?)\, \omega$, the two new actions a_2 and a_3 as well as five states namely s_5, s_6, s_7, s_8 and s_9 are introduced (for the simplicity the actions' specification is omitted). Assume that in the state s_1 Ann wants to change a degree of John's belief about the thesis p. Before she executes an arbitrary sequence of the actions a_2, a_3, she considers two doxastic alternatives: the state s_1, where John believes p with the degree $\frac{1}{10}$, i.e., $M!^{1,10}_{John}p$ and the state s_4, where John believes p with the degree $\frac{1}{4}$, i.e., $M!^{1,4}_{John}p$. Therefore, Ann predicts the argumentation's results beginning from the states s_1 and s_4 - see Fig. 4(a).

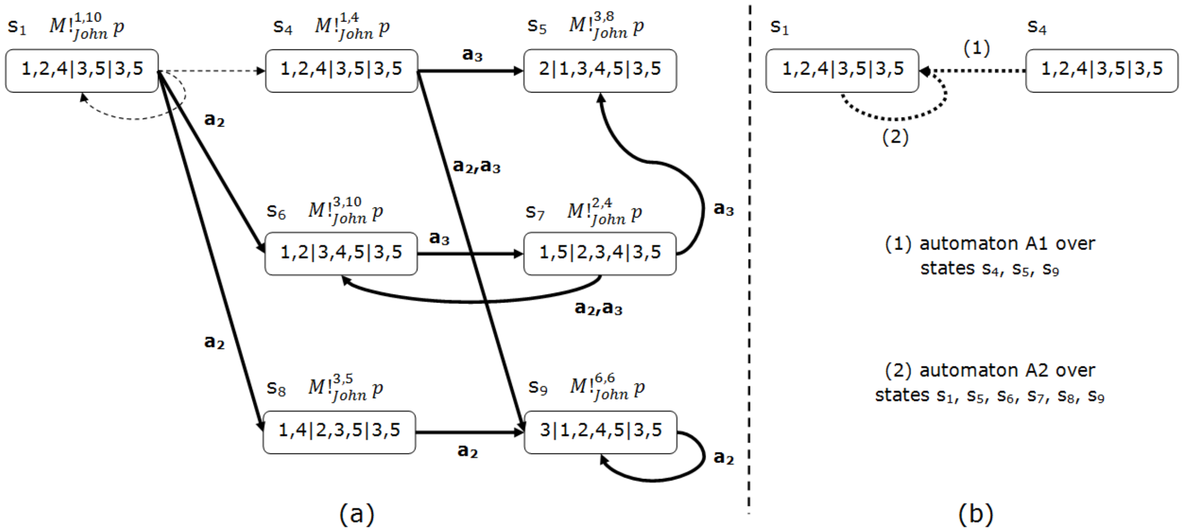

Figure 4. (a) An example model $\mathcal{M}$. Doxastic relation of Ann is marked with broken lines, actions a_2, a_3 done by Ann are marked with bolded solid lines. (b) The solutions for the subquestions $\left(\mathcal{M}, s_1, \Diamond (Ann :?)\, M!^3_{John}p\right)$ and $\left(\mathcal{M}, s_4, \Diamond (Ann :?)\, M!^3_{John}p\right)$.

Now we study the input question $\left(\mathcal{M}, s_1, M^0_{Ann}\Diamond (Ann :?)\, M!^3_{John}p\right)$. If the operator M^0_{Ann} is applied in the state s_1 of the model $\mathcal{M}$, then the following states are reached – s_1, s_4. Next, for these two states the unknown $\Diamond (Ann :?)\, M!^3_{John}p$ is investigated, i.e. subquestions $\left(\mathcal{M}, s_1, \Diamond (Ann :?)\, M!^3_{John}p\right)$ and $\left(\mathcal{M}, s_4, \Diamond (Ann :?)\, M!^3_{John}p\right)$ are studied. In order to find their solutions, two nondeterministic automata are constructed, the automaton $A1$ and the automaton $A2$ respectively – see Fig. 4(b).

According to the standard structure of a nondeterministic finite automaton $A = (Q, \Sigma, \delta, s, F)$, where Q is a finite set of states, Σ is a finite set of input symbols, $\delta \subseteq (Q \times \Sigma) \times Q$ is a transition relation, $s \in Q$ is an automaton initial state and $F \subseteq Q$ is a set of accept states, the automata being considered can be defined as follows (see also Fig. 5(a)):

- $A1 = (Q_1, \Sigma_1, \delta_1, s_4, F_1) = (\{s_4, s_5, s_9\} \cup \{\_\}, \{a_2, a_3\}, \delta_1, s_2, \{s_3\})$ and "_" is a dummy state, where

$$\begin{aligned}\delta_1 \;\; = \;\; & \{((s_4, a_2), s_9), ((s_4, a_3), s_5), ((s_4, a_3), s_9), ((s_9, a_2), s_9), \\ & \ldots, ((\cdot, \cdot), \_) \text{ in the other case}\},\end{aligned}$$

- $A2 = (Q_2, \Sigma_2, \delta_2, s_1, F_2) = (\{s_1, s_5, s_6, s_7, s_8, s_9\} \cup \{\_\}, \{a_2, a_3\}, \delta_2, s_1, \{s_5, s_6, s_8\})$, where

$$\begin{aligned}\delta_2 \;=\; & \{((s_1, a_2), s_6), ((s_1, a_2), s_8), ((s_6, a_3), s_7), ((s_7, a_2), s_6), \\ & ((s_7, a_3), s_5), ((s_7, a_3), s_6), ((s_8, a_2), s_9), ((s_9, a_2), s_9), \\ & \ldots, ((\cdot, \cdot), \_) \text{ in the other case}\},\end{aligned}$$

Thus, $\mathcal{M}, s_4 \models \Diamond (Ann : x_{s_4}) M!^3_{John} p$ holds for every x_{s_4} such, that x_{s_4} is a sequence of symbols, splitted with a comma symbol, which is accepted by the automaton $A1$, e.g. $x_{s_4} = (a_3)$. Similarly, $\mathcal{M}, s_1 \models \Diamond (Ann : x_{s_1}) p$ holds for every x_{s_1} such, that x_{s_1} is a sequence of symbols splitted with a comma symbol, which is accepted by the automaton $A2$, e.g. $x_{s_1} = (a_2, a_3, a_3, a_3, a_2)$.

Finally, automata $A1$ and $A2$ are bounded with respect to the operator M^0_{Ann}. In this case the product automaton $A3 = (Q_3, \Sigma_3, \delta_3, (s_4, s_1), F_3)$ is constructed, where $Q_3 = (Q_1 \times Q_2) \cup \{\_\}$, $\Sigma_3 = \Sigma_1 \cup \Sigma_2$, $F_3 = \{(x, y) \in Q_3 \setminus \{\_\} : f((x, y)) > 0\}$ for $f : (Q_3 \setminus \{\_\}) \to \mathbb{N}$ such, that

$$f((x, y)) \;=\; \begin{cases} 0 & \text{for } x \notin F_1 \wedge y \notin F_2 \\ 1 & \text{for } (x \in F_1 \wedge y \notin F_2) \vee (x \notin F_1 \wedge y \in F_2) \\ 2 & \text{for } x \in F_1 \wedge y \in F_2 \end{cases}.$$

It is important to mark that the definition of the set F_3 is strongly related to a bounding operator, $f((x, y)) > 0$ used because M^0_{Ann} (more than 0) is an outer modality. Hence, if M^1_{Ann} is considered, then $f((x, y)) > 1$ is obtained. The transition relation δ_3 is described below (see also Fig. 5(b))

$$\begin{aligned}\delta_3 \;=\; & \{(((s_4, s_1), a_2), (s_9, s_6)), (((s_4, s_1), a_2), (s_9, s_8)), (((s_4, s_1), a_3), (s_5, \_)), \\ & (((s_4, s_1), a_3), (s_9, \_)), (((s_9, s_6), a_2), (s_9, \_)), (((s_9, s_6), a_3), (\_, s_7)), \\ & (((s_9, s_8), a_2), (s_9, s_9)), (((s_9, \_), a_2), (s_9, \_)), (((\_, s_7), a_2), (\_, s_6)), \\ & (((\_, s_7), a_3), (\_, s_5)), (((\_, s_7), a_3), (\_, s_6)), (((\_, s_6), a_3), (\_, s_7)), \\ & \ldots, (((\cdot, \cdot), \cdot), \_) \text{ in the other case}\}.\end{aligned}$$

Therefore, an answer to the input question $\left(\mathcal{M}, s_1, M^0_{Ann} \Diamond (Ann :?) M!^3_{John} p\right)$ is $true$ if $? \in X$, where X is set of all sequence of symbols, splitted with a comma symbol, accepted by the product automaton $A3$, e.g $? = (a_2)$, $? = (a_3)$ or $? = (a_2, a_3, a_2, a_3, a_2)$.

If the input question is $\left(\mathcal{M}, s_1, M^1_{Ann} \Diamond (Ann :?) M!^3_{John} p\right)$ then the same reasoning is repeated. Only, the definition of the accept states set for the product automaton $A3$ is different. Now, $F_3 = \{(x, y) \in Q_3 \setminus \{\_\} : \alpha((x, y)) > 1\}$ thus, $F_3 = \emptyset$. So, there is no sequence of symbols (argumentation), which is accepted by the automaton $A3$. Therefore, a ? symbol has no proper valuation, i.e. $\mathcal{M}, s_1 \not\models M^1_{Ann} \Diamond (Ann : P) M!^3_{John} p$ for all $P \in \Pi$. In this case the answer to the question is $false$.

7. Conclusions and future work

The general motivation of our research is to provide the framework for representing as well as the tools for analyzing the persuasion process. In particular, we are interested in studying those **properties of persuasion** which are typically not considered in formal approaches, but which are very important from the perspective of the **real-life practice**. At this stage of our research we include in our model the aspects

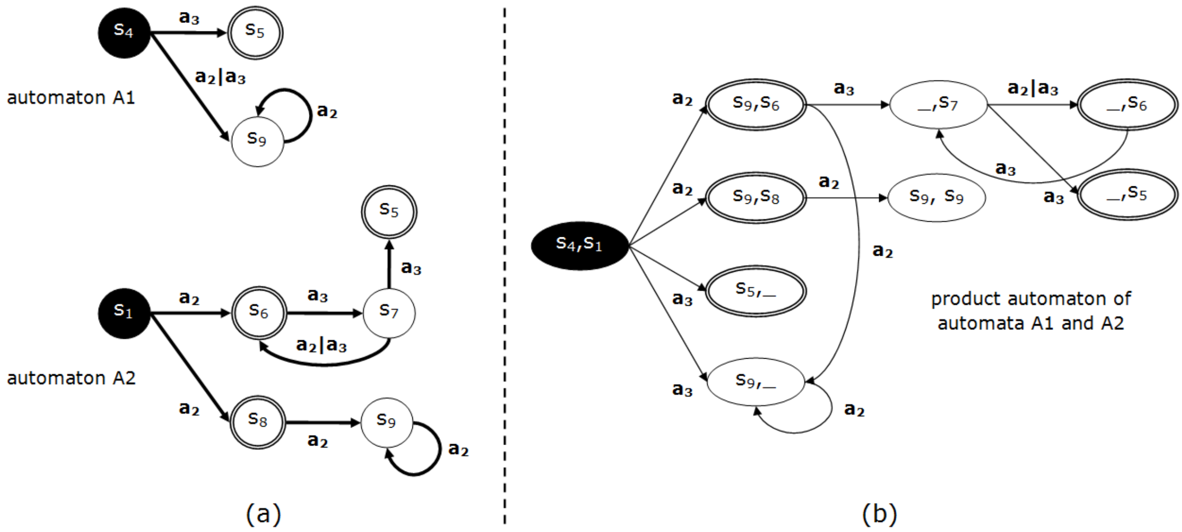

Figure 5. (a) The automata $A1$ and $A2$. The initial states are filled up with black color, the accept states are marked with double lines. For simplicity all dummy states are omitted. (b) A product automaton $A3$ of automata $A1$ and $A2$ bounded with respect to the operator $M^0_{ag_1}$.

related to the fact that some arguments have stronger impact on beliefs than the other ones. Therefore, we want Perseus to investigate strength, dynamics and subjectivity of persuasion. It enables to study *degree of changes* in agent's beliefs generated by the persuasion and to *track those changes* in the agent's beliefs step-by-step, i.e. at any intermediate stage of the persuasion (after the first argument, after the second and so on). Moreover, Perseus allows to take into account the *subjective aspects* of persuasion. It shows what impact the proponent and audience have on a persuasion process and its effects.

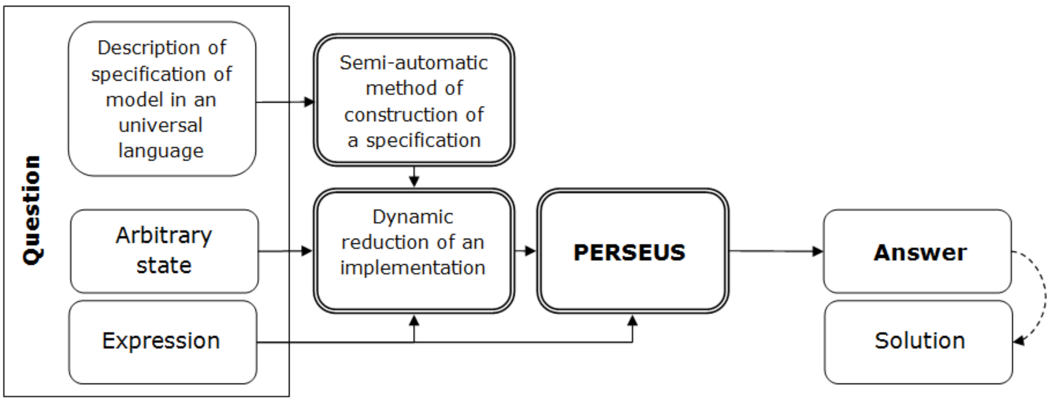

Figure 6. The future development of the Perseus project.

The future directions of the research on the Perseus computer system are focused on the following problems: (a) determining the computational complexity of the described methods of solving an unknowns in an arbitrary class of the models of the multi-agent systems, (b) optimizing the argumentations with respect to an arbitrary measure, e.g. a length of an argumentation or a cost of an argumentation, (c) answering questions, in which the expressions are consisted of more than one nested unknowns, e.g. $M_i^? B_j^? \omega$. In the last case an optimization of a function of beliefs' dependencies is possible, if an arbitrary measure is introduced. We also do a research on the selected aspects of the complexity of a model. It is known that the real-life applications of $\mathcal{AG}_n$ logic formalism lead to an implementation of a model, which size is at least exponential relatively to an arbitrary property of a multi-agent system. Therefore, our future work is also dedicated to the next two problems – see Fig. 6. The first one is to design a supporting method of semi-automatic construction of a specification of a model. In order to complete this task an universal language of describing an arbitrary class of the specifications of the models is necessary. The second problem is a dynamic reduction of a model implementation with respect to an input expression ϕ. In this case an implementation of a model can be restricted only to the states of a model $\mathcal{M}$ as well as the agents and the actions, which are necessary from a point of view of an input question $(\mathcal{M}, s, \phi)$.

We want to use our studies in the domains like marketing, e-business, e-commerce, e-negotiations. Perseus may find applications in examining the issues of influencing the beliefs of customers (e.g. stimulating, realizing, creating their needs) or analyzing the behavior of customers in order to win new clients and make them buy more.

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